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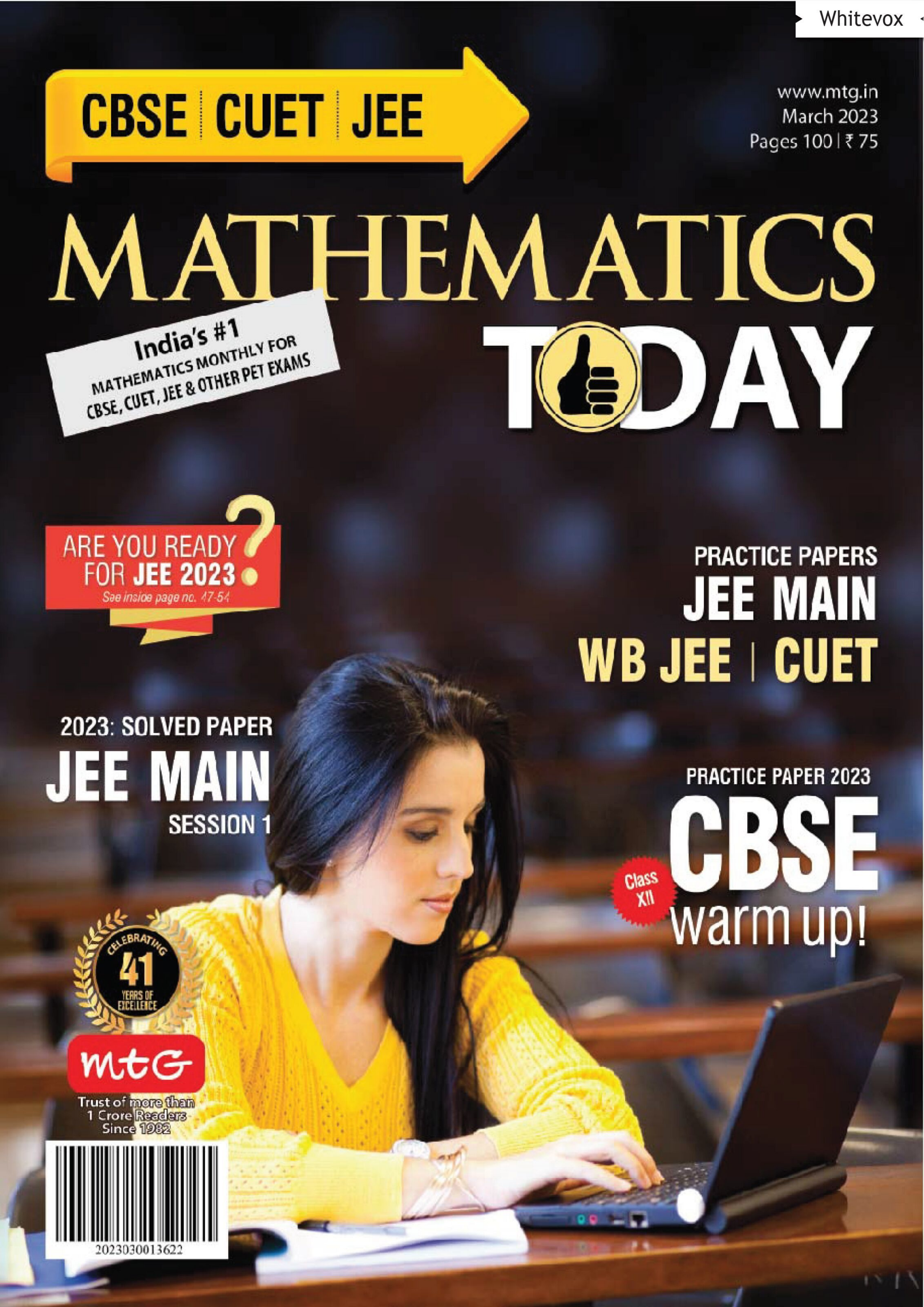

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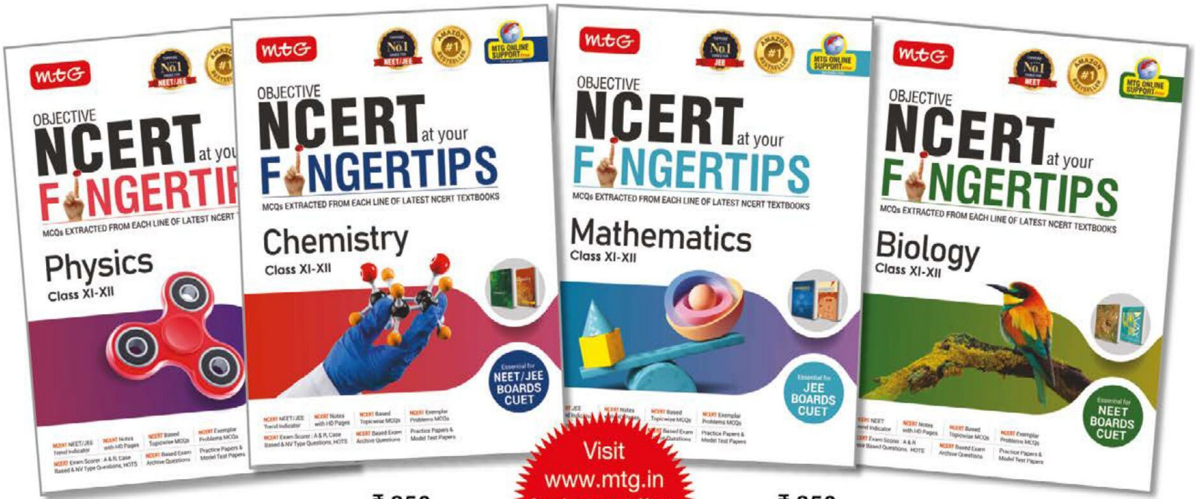

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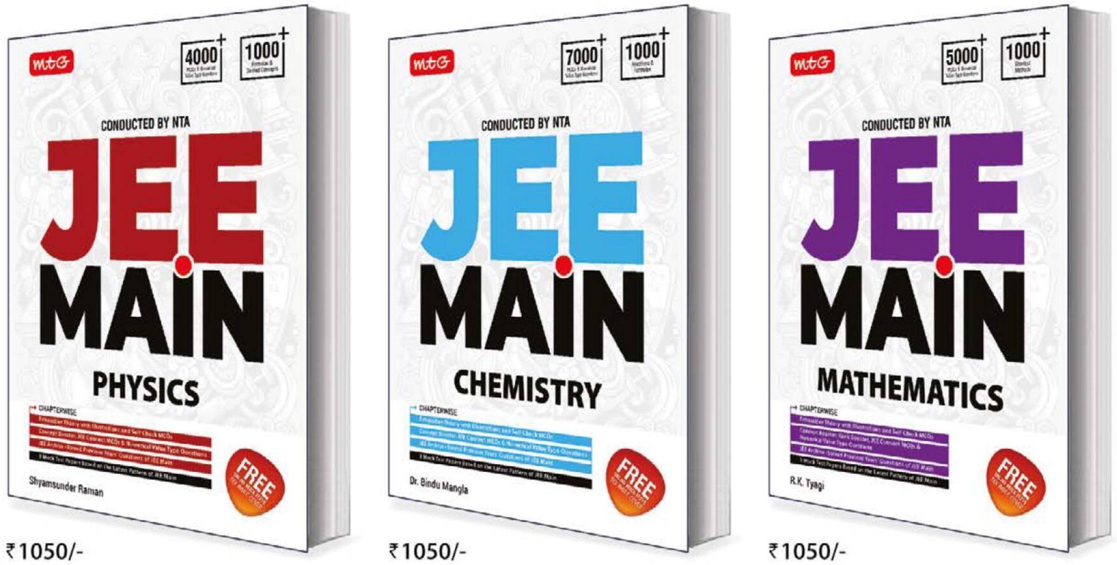
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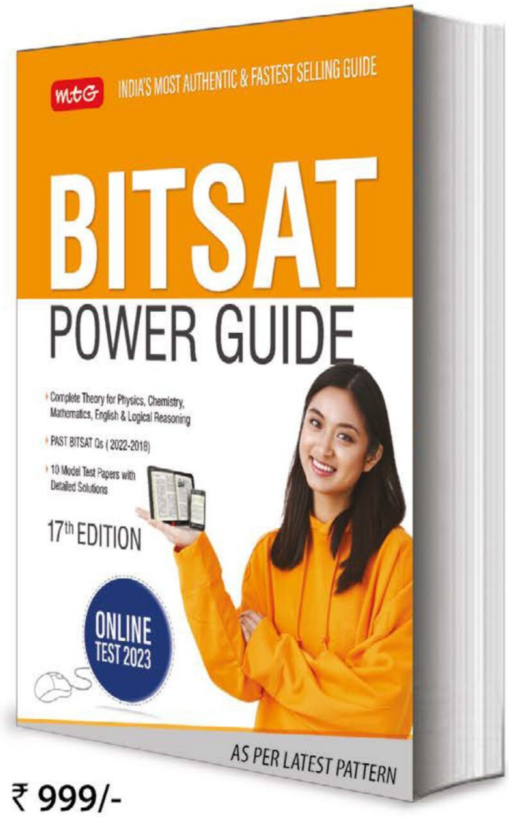
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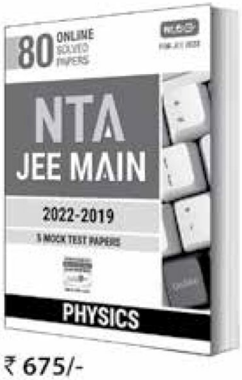
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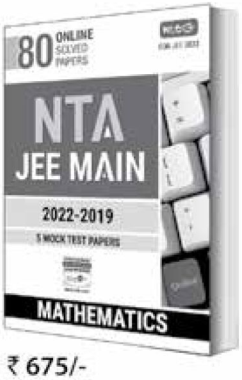
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SOLVED PAPER

Held on
24th Jan, Morning Shift
(Session - 1)

JEE MAIN 2023

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1)\,dx = 0, x > 0,\ y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to

(a) $2 - e$ (b) e (c) 1 (d) 3

2. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is

(a) $\frac{23}{3}$ (b) $\frac{22}{3}$ (c) 9 (d) $\frac{25}{3}$

3. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements

(S1) : If $P(A) = 0$, then $A = \phi$

(S2) : If $P(A) = 1$, then $A = \Omega$

Then

(a) both (S1) and (S2) are true

(b) only (S1) is true

(c) only (S2) is true

(d) both (S1) and (S2) are false

4. The value of $\sum_{r=0}^{22} {}^{22}C_r\, {}^{23}C_r$ is

(a) ${}^{45}C_{24}$ (b) ${}^{45}C_{23}$ (c) ${}^{44}C_{23}$ (d) ${}^{44}C_{22}$

5. Let α be a root of the equation

$(a - c)x^2 + (b - a)x + (c - b) = 0$

where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular. Then, the value of

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$$ is

(a) 6 (b) 3 (c) 9 (d) 12

6. $\lim\limits_{t \to 0} \left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}} \right)^{\sin^2 t}$ is equal to

(a) n^2 (b) $n^2 + n$ (c) n (d) $\frac{n(n+1)}{2}$

7. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(a) $20\sqrt{2}$ (b) 31 (c) $13\sqrt{2}$ (d) 26

8. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

has unique solution is $k/6$, then the sum of value of k and all possible values of N is

(a) 18 (b) 19 (c) 20 (d) 21

9. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then at $x = 0$

(a) f is continuous but not differentiable

(b) f is continuous but f' is not continuous

(c) f' is continuous but not differentiable

(d) f and f' both are continuous

10. Let PQR be a triangle. The points A, B and C are the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{Area(\Delta PQR)}{Area(\Delta ABC)}$ is equal to

(a) 4 (b) 3 (c) 2 (d) 5/2

11. For three positive intgers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2 r}$ and $r = pq + 1$ such that 3, $3 \log_y x$, $3 \log_z y$, $7\log_x z$ are in A.P. with common difference 1/2. The $r-p-q$ is equal to

(a) 6 (b) 2 (c) 12 (d) –6

12. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

13. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has

(a) no solution
(b) exactly two solutions in $(-\infty, \infty)$
(c) a unique solution in $(-\infty, 1)$
(d) a unique solution in $(-\infty, \infty)$

14. Let $p, q \in R$ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq), i=\sqrt{-1}$. The $p + q + q^2$ and $p - q + q^2$ are roots of the equation

(a) $x^2 - 4x + 1 = 0$ (b) $x^2 + 4x + 1 = 0$
(c) $x^2 - 4x - 1 = 0$ (d) $x^2 + 4x - 1 = 0$

15. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2B$, then

(a) $A^2 = I$ or $B = I$ (b) $A^2B = BA^2$
(c) $AB = I$ (d) $A^2B = I$

16. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

(a) length of latus rectum 3/2
(b) length of latus rectum 2
(c) directrix $4x = 3$
(d) directrix $4x = -3$

17. The distance of the point (7, –3, –4) from the plane passing through the points (2, –3, 1), (–1, 1, –2) and (3, –4, 2) is

(a) $5\sqrt{2}$ (b) $4\sqrt{2}$ (c) 4 (d) 5

18. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(a) $((\sim P \vee Q) \wedge ((\sim Q) \vee P)$
(b) $(\sim Q) \vee P$
(c) $((\sim P) \vee Q) \wedge (\sim Q)$ (d) $(\sim P) \vee Q$

19. The relation $R = \{(a, b) : gcd(a, b) = 1, 2a \neq b, a, b, \in Z\}$ is

(a) reflexive but not symmetric
(b) transitive but not reflexive
(c) symmetric but not transitive
(d) neither symmetric nor transitive

20. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v}\cdot\vec{w} = 2$ and $\vec{v}\times\vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}\cdot\vec{w}$ is equal to

(a) 1 (b) 2 (c) 3/2 (d) –2/3

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. Let $\lambda \in R$ and let the equation E be $|x^2| - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of $E\}$ is _______.

22. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is _______.

23. The value of $\frac{8}{\pi}\int_0^{\pi/2}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is _______.

24. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to _______.

25. The 4th term of GP is 500 and its common ratio is $1/m$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is _______.

26. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to _______.

27. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _______.

28. Suppose $\sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _______.

29. The value of $12\int_0^3 |x^2 - 3x + 2|\,dx$ is _______.

30. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _______.

SOLUTIONS

1. **(c):** Given, $x^3dy + (xy - 1)dx = 0$

$\Rightarrow x^3dy = (1 - xy)dx$

$\Rightarrow \frac{dy}{dx}=\frac{1-xy}{x^3} \Rightarrow \frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$, which is linear, D.E.

Now, $\text{I.F.}=e^{\int p\,dx}=e^{\int \frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

Solution is given by $ye^{-1/x}=\int \frac{1}{x^3}e^{-1/x}\,dx+c$

Putting $u=\frac{-1}{x} \Rightarrow du=\frac{1}{x^2}dx$, we get

$ye^{-1/x}=\int -e^u u\,du+c=-[ue^u-\int e^u du]+c$

$= -[ue^u - e^u] + c = (1 - u)e^u + c$

$\Rightarrow ye^{-1/x}=\left(1+\frac{1}{x}\right)e^{-1/x}+c$ ...(i)

Put $x=\frac{1}{2}$ and $y=3-e$; $3e^{-2}-e^{-1}=3e^{-2}+c \Rightarrow c=-e^{-1}$

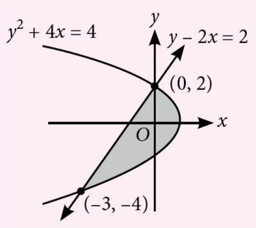
Equation (i) becomes, $ye^{-1/x}=e^{-1/x}+\frac{e^{-1/x}}{x}-e^{-1}$

Put $x = 1$ to get $y(1)$,

$\Rightarrow ye^{-1}=e^{-1}+e^{-1}-e^{-1}=e^{-1} \Rightarrow y=1$

2. **(c):** Given curves are $y^2 + 4x = 4$ and $y - 2x = 2$

The points of intersection of the given curves are $(0, 2)$ and $(-3, -4)$.



Required area

$=\int_{-4}^{2}\left[\left(\frac{4-y^2}{4}\right)-\left(\frac{y-2}{2}\right)\right]dy$

$=\int_{-4}^{2}\frac{-y^2-2y+8}{4}dy=\frac{1}{4}\left[-\frac{y^3}{3}-y^2+8y\right]_{-4}^{2}$

$=\frac{1}{4}\left[\left(-\frac{8}{3}-4+16\right)-\left(\frac{64}{3}-16-32\right)\right]$

$=\frac{1}{4}\left[\left(\frac{28}{3}\right)-\left(-\frac{80}{3}\right)\right]=\frac{1}{4}\left(\frac{108}{3}\right)=\frac{108}{12}=9$ sq. units

3. **(a):** Ω be the Sample space A be an event.

If $P(A) = 0 \Rightarrow A = \phi$

If $P(A) = 1 \Rightarrow A = \Omega$

Then both statements are true.

4 **(b) :** $\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_r=\sum_{r=0}^{22}{}^{22}C_r\,{}^{23}C_{23-r}$

$={}^{22+23}C_{23}={}^{45}C_{23}$

5. **(b) :** Given matrix is singular,

$$\therefore \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}=0$$

$\Rightarrow \alpha^2(c - b) - \alpha(c - a) + 1\,(b - a) = 0$

It is singular if $\alpha = 1 \Rightarrow c-b-c+a+b-a=0$

Now, $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$

$=\frac{(a-c)^3}{(a-c)(b-a)(c-b)}+\frac{(b-a)^3}{(a-c)(b-a)(c-b)}$

$+\frac{(c-b)^3}{(a-c)(b-a)(c-b)}$

$=\frac{(a-c)^3+(b-a)^3+(c-b)^3}{(a-c)(b-a)(c-b)}$

$=\frac{3(a-c)(b-a)(c-b)}{(a-c)(b-a)(c-b)}=3 \quad (\because a-c+b-a+c-b=0)$

6. **(c):** Let $L=\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+\ldots+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$

$$\Rightarrow L=\lim_{t\to 0} n\left[\left(\frac{1}{n}\right)^{\text{cosec}^2 t}+\left(\frac{2}{n}\right)^{\text{cosec}^2 t}+\ldots+\left(\frac{n-1}{n}\right)^{\text{cosec}^2 t}+1\right]^{\sin^2 t}$$

$\Rightarrow L=n(1^{\sin^2(0)})=n(1)=n$

$$\left[0<\frac{k}{n}<1,\left(\frac{k}{n}\right)^{\infty}=0 \text{ for } 1\le k<n\right]$$

7. **(d) :** Equation of line passing through the point $(-1, 9, -16)$ and parallel to line $\frac{x+4}{3}=\frac{2-y}{4}=\frac{z-3}{12}$ is $\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}=\lambda$

Any point on this line $= (3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$

Point of intersection line and plane

$2(3\lambda - 1) + 3(-4\lambda + 9) - 1(12\lambda - 16) = 5$

$\Rightarrow 6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$

$\Rightarrow \lambda = 2 \quad \therefore$ Point $= (5, 1, 8)$

$\therefore$ Distance $=\sqrt{(5+1)^2+(9-1)^2+(-16-8)^2}$

$=\sqrt{36+64+576}=26$ units

8. **(c):** For unique solution, determinant $\neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}\neq 0$$

$\Rightarrow N^2 - 5N + 6 \neq 0 \Rightarrow (N-2)(N-3) \neq 0 \Rightarrow N \neq 2, 3$

$\therefore$ Possible values of N are $1, 4, 5, 6$ and $\frac{k}{6} = \frac{4}{6} \Rightarrow k = 4$

Required sum = 1 + 4 + 5 + 6 + 4 = 20

9. (b) : $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\text{LHD} = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0^-} -\frac{h^2 \sin(1/h)}{-h} = 0$$

$$\text{RHD} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 \sin(1/h)}{h} = 0$$

$\Rightarrow f(x)$ is continuous and differentiate at $x = 0$

Now, $f'(x) = \begin{cases} 2x \sin(1/x) - \cos(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x \to 0} f'(x) = \lim_{x \to 0} \left[2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) \right] = 0 - [-1, 1] \neq 0$$

$f'(0) = 0$

Hence, $f'(x)$ is discontinuous at $x = 0$.

10. (b) : Let the position vector of P, Q, R be $\vec{0}, \vec{a}, \vec{b}$

$\Rightarrow$ P.V. of $A = \frac{2\vec{a} + \vec{b}}{3}$, P.V. of $B = \frac{2\vec{b}}{3}$

and P.V. of $C = \frac{\vec{a}}{3}$

$$\therefore \overrightarrow{AB} = \frac{2\vec{b}}{3} - \left(\frac{2\vec{a} + \vec{b}}{3}\right) = \frac{\vec{b}}{3} - \frac{2\vec{a}}{3} = \frac{\vec{b} - 2\vec{a}}{3}$$

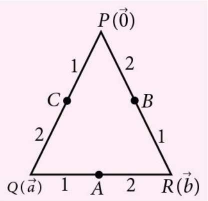
$$\overrightarrow{CA} = \frac{2\vec{a} + \vec{b}}{3} - \frac{\vec{a}}{3} = \frac{\vec{a} + \vec{b}}{3}$$

Area of $\Delta PQR = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{1}{2}|\vec{a} \times \vec{b}|$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{CA} \times \overrightarrow{AB}\right|$

$$= \frac{1}{2}\left|\left(\frac{\vec{a} + \vec{b}}{3}\right) \times \left(\frac{\vec{b} - 2\vec{a}}{3}\right)\right| = \frac{1}{2}\left|\frac{\vec{a} \times \vec{b}}{3}\right|$$

$$\therefore \frac{\text{Area } (\Delta PQR)}{\text{Area } (\Delta ABC)} = \frac{\frac{1}{2}|\vec{a} \times \vec{b}|}{\frac{1}{2}\left|\frac{\vec{a} \times \vec{b}}{3}\right|} = 3$$



11. (b) : $pq^2 \log x = qr \log y = p^2 r \log z$

Now, $\frac{3 \log x}{\log y} - 3 = \frac{1}{2} \Rightarrow \frac{\log x}{\log y} = \frac{7}{6} = \frac{qr}{pq^2} = \frac{r}{pq}$

$\Rightarrow 7pq = 6r \Rightarrow 7(r-1) = 6r \Rightarrow r = 7$

and $\frac{3 \log y}{\log z} - 3 = 1 \Rightarrow \frac{\log y}{\log z} = \frac{4}{3} = \frac{p^2 r}{qr} = \frac{p^2}{q} \Rightarrow 3p^2 = 4q$

Also, $\frac{7 \log z}{\log x} - 3 = \frac{3}{2} \Rightarrow \frac{7 \log z}{\log x} = \frac{9}{2} = \frac{7pq^2}{p^2 r} = \frac{7q^2}{pr}$

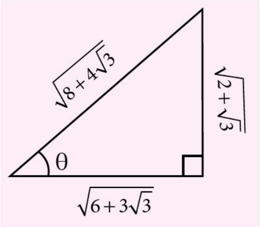
$\Rightarrow \frac{7q^2}{p \times 7} = \frac{9}{2} \Rightarrow \frac{q^2}{p} = \frac{9}{2} \Rightarrow 2q^2 = 9p \Rightarrow 4q^4 = 81p^2$

$\Rightarrow 4q^4 = 27 \times 3p^2 = 27 \times 4q \Rightarrow q^3 = 27 \Rightarrow q = 3$

We have, $r = pq + 1 \Rightarrow 7 = 3p + 1 \Rightarrow p = 2$

$\therefore r - p - q = 7 - 2 - 3 = 2$

12. (d) : $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$



$$= \tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}\right) + \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{6+3\sqrt{3}}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3})}\right) + \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{3(2+\sqrt{3})}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3})}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$$

13. (d) : $x^2 - 4x + [x] + 3 = x[x]$

$\Rightarrow x^2 - x[x] - (x - [x] + 3(x - 1) = 0$

$\Rightarrow x(x - [x]) - 1(x - [x]) - 3(x - 1) = 0$

$\Rightarrow (x - 1)(x - [x]) - 3(x - 1) = 0$

$\Rightarrow (x - 1)(x - [x] - 3) = 0$

$\Rightarrow (x - 1)(\{x\} - 3) = 0 \Rightarrow x = 1$ as $\{x\} \neq 3$

14. (a): Given, $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$

$\Rightarrow (2e^{-i\pi/3})^{200} = 2^{199}(p + iq)$

$$\Rightarrow 2^{200}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^{200} = 2^{199}(p + iq)$$

$$\Rightarrow 2\left(\cos\frac{200\pi}{3} - i\sin\frac{200\pi}{3}\right) = p + iq$$

$\therefore p = -1, q = -\sqrt{3}$

$\Rightarrow p + q + q^2 = -1 - \sqrt{3} + 3 = 2 - \sqrt{3}$

and $p - q + q^2 = -1 + \sqrt{3} + 3 = 2 + \sqrt{3}$

Sum of root $= 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Product of root $= (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Required quadratic equation is $x^2 - 4x + 1 = 0$

15. (b) : $A^2 + B = A^2B$

$\Rightarrow A^2 + B - A^2B = 0 \Rightarrow A^2 - A^2B + B = 0$

$\Rightarrow A^2(\mathrm{I} - B) + B = 0 \Rightarrow A^2(I - B) + B - I = -I$

$\Rightarrow (A^2 - I)(I - B) = -I \Rightarrow (A^2 - I)(B - I) = I$

$\Rightarrow A^2B - A^2 - B = 0 \Rightarrow A^2(B - I) = B$

$\Rightarrow A^2 = B(B - I)^{-1} \Rightarrow A^2 = B(A^2 - I)$

$\Rightarrow A^2 = BA^2 - B \Rightarrow A^2 + B = BA^2 \Rightarrow A^2B = BA^2$

16. (c): Let the equation of tangent to $y^2 = 24x$ is $ty = x + 6t^2$

Now, $ty = x + 6t^2$ meet the curve $xy = 2$ at point A and B.

Let mid-point of AB be (h, k)

$$ty = \frac{2}{y} + 6t^2 \qquad \left[\because\ x = \frac{2}{y}\right]$$

$ty^2 - 6t^2y - 2 = 0\,;\ y_1 + y_2 = 6t$

$$t \cdot \frac{2}{x} = x + 6t^2 \qquad \left[\because\ y = \frac{2}{x}\right]$$

$x^2 + 6t^2x - 2t = 0\,;\ x_1 + x_2 = -6t^2$

Mid point P is $(-3t^2, 3t)$

$$\Rightarrow h = -3t^2, k = 3t \Rightarrow \left(\frac{h}{-3}\right) = \left(\frac{k}{3}\right)^2 \Rightarrow y^2 = -3x$$

17. (a): Let $A(2, -3, 1)$, $B(-1, 1, -2)$ and $C(3, -4, 2)$

Cartesian equation of plane passing through A, B and C is

$$\begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow (x - 2)(4 - 3) - (y + 3)(-3 + 3) + (z - 1)(3 - 4) = 0$

$\Rightarrow x - 2 - z + 1 = 0 \Rightarrow x - z = 1$

$$\text{Required distance} = \left|\frac{1(7) - (-4) - 1}{\sqrt{1+1}}\right| = \frac{10}{\sqrt{2}} = 5\sqrt{2}\ \text{sq. units}$$

18. (a): Constructing the truth table.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P \wedge Q$	$\sim(P \wedge Q) \vee (\sim P \wedge Q)$	$(\sim P) \wedge (\sim Q)$	$(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$
T	T	F	F	T	F	F	F	F	T
T	F	F	T	F	T	F	T	F	F
F	T	T	F	F	T	T	T	F	F
F	F	T	T	F	T	F	T	T	T

P	Q	$(\sim P) \vee Q$	$(\sim Q) \vee P$	$((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Thus, the statement given in question is equivalent to $((\sim P \vee Q) \wedge ((\sim Q) \vee P)$

19. (d) : Reflexive : $gcd\,(a, a) = a \neq 1$ for $a \neq 1$.

Symmetric : Let $a = 2$, $b = 1$; $gcd\,(a, b) = 1$, $b \neq 2a$

Thus, $(2, 1) \in R$; $gcd\,(1, 2) = 1$. But $2 = 2 \times 1$.

Therefore $(1, 2) \notin R$. Hence, R is not symmetric.

Transitive : $(2, 3) \in R$; $(3, 8) \in R$

But $(2, 8) \notin R$. Hence R is not transitive

R is neither symmetric nor transitive

20. (a): $\vec{v} \times \vec{w} = (\vec{u} + \lambda\vec{v}) = \hat{i} - \hat{j} - 2\hat{k} + \lambda(2\hat{i} + \hat{j} - \hat{k})$

$= (2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} - (2 + \lambda)\hat{k}$

Now, $\vec{v} \cdot (\vec{v} \times \vec{w}) = 0 \Rightarrow (2\hat{i} + \hat{j} - \hat{k}) \cdot ((2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} - (\lambda + 2)\hat{k}) = 0$

$\Rightarrow 2(2\lambda + 1) + \lambda - 1 + \lambda + 2 = 0 \Rightarrow 6\lambda + 3 = 0 \Rightarrow \lambda = -\dfrac{1}{2}$

$\vec{w} \cdot (\vec{v} \times \vec{w}) = \vec{w} \cdot (\vec{u} + \lambda\vec{v}) = \vec{u} \cdot \vec{w} + \lambda\vec{v} \cdot \vec{w} = 0$

$\vec{u} \cdot \vec{w} = -\lambda\vec{v} \cdot \vec{w}\ ;\ \vec{u} \cdot \vec{w} = \dfrac{1}{2} \times 2 = 1$

21. (5): $|x|^2 - 2|x| + |\lambda - 3| = 0$

$(|x| - 1)^2 + |\lambda - 3| - 1 = 0$ ...(i)

Given that x is an integer $\Rightarrow |x| - 1$ is an integer

$(|x| - 1)^2 - 1$ is also an integer. Thus $|\lambda - 3|$ is also an integer.

Therefore λ is also an integer, $(|x| - 1)^2 \leq 1$

Case I : $|x| - 1 = 0 \Rightarrow x = \pm 1 \quad \therefore\ |\lambda - 3| = 1 \Rightarrow \lambda = 4, 2$

Possible values of $\lambda + x = 5, 3, 1$.

Case II : $|x| - 1 = 1 \Rightarrow x = \pm 2 \quad \therefore\ |\lambda - 3| = 0 \Rightarrow \lambda = 3$

Possible values of $x + \lambda = 5, 1$

Considering all possible cases maximum value of $\lambda + x = 5$

22. (546) : 5 language courses 7 non language courses

He can select :

(i) 0 language courses and 5 non language courses in ${}^5C_0 \times {}^7C_5 = 21$ ways

(ii) 1 language courses and 4 non language courses in ${}^5C_1 \times {}^7C_4 = 175$ ways

(iii) 2 language courses and 3 non language courses in ${}^5C_2 \times {}^7C_3 = 350$ ways

Required number of ways = 21 + 175 + 350 = 546 ways.

23. (2): Let $I = \displaystyle\int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}}\,dx$ ...(i)

$$I = \int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}\,dx \qquad \text{...(ii)}$$

$$\left[\because\ \int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx\right]$$

Adding equation (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \Rightarrow \frac{8}{\pi}I = 2$$

24. (14): $L_1 : \frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}; L_2 : \frac{x-6}{3}=\frac{y-1}{-2}=\frac{z+8}{0}$

$$\text{Shortest distance} = \frac{\left|\begin{vmatrix} 6-2 & 1-(-1) & -8-6 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}\right|}$$

$$= \frac{\left|\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}\right|}{|4\hat{i}+6\hat{j}-12\hat{k}|} = \frac{4(0+4)-2(0-6)-14(-6-6)}{\sqrt{4^2+6^2+12^2}}$$

$$= \frac{16+12+14\times 12}{14} = 14 \text{ units.}$$

25. (12): Let the first term be A

$T_4 = 500$; $\frac{A}{m^3} = 500$; $S_6 - S_5 > 1 \Rightarrow T_6 > 1 \Rightarrow \frac{A}{m^5} > 1$

$\Rightarrow \frac{500}{m^2} > 1 \Rightarrow m^2 < 500 \Rightarrow m < \sqrt{500} \Rightarrow m \le 22$

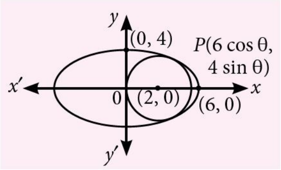
$(\because m \in N)$

$S_7 - S_6 < \frac{1}{2} \Rightarrow T_7 < \frac{1}{2} \Rightarrow \frac{A}{m^6} < \frac{1}{2} \Rightarrow \frac{500}{m^3} < \frac{1}{2}$

$m^3 > 1000 \Rightarrow m \ge 11$

Thus, $11 \le m \le 22$ $\therefore$ m can take 12 values

26. (118) : Let P be the point where ellipse and circle touch each other.



Let P be $(6\cos\theta, 4\sin\theta)$

Equation of tangent to ellipse at point P is

$$\frac{x\cos\theta}{6}+\frac{y\sin\theta}{4}=1$$

$$\Rightarrow y = \frac{4}{\sin\theta}\left(\frac{-x\cos\theta}{6}+1\right) \Rightarrow y = -\frac{2\cos\theta}{3\sin\theta}x+\frac{4}{\sin\theta}$$

Let $m_1 = \frac{-2\cos\theta}{3\sin\theta}$.

Slope of normal to circle at $P = m_2$.

$m_2 = \frac{(4\sin\theta - 0)}{(6\cos\theta - 2)}$; $m_1 m_2 = -1$

$$\Rightarrow \frac{-2\cos\theta}{3\sin\theta} \times \frac{4\sin\theta}{(6\cos\theta-2)} = -1 \Rightarrow 8\cos\theta = 3(6\cos\theta - 2)$$

$$\Rightarrow 6 = 10\cos\theta \Rightarrow \cos\theta = \frac{3}{5} \Rightarrow \sin\theta = \frac{4}{5} \text{ So, } P \equiv \left(\frac{18}{5}, \frac{16}{5}\right)$$

$$r^2 = \left(\frac{18}{5}-2\right)^2 + \left(\frac{16}{5}\right)^2 = (1-2)^2 + \alpha^2 = \frac{64}{25}+\frac{256}{25} = 1+\alpha^2$$

$$\alpha^2 = \frac{320}{25} - 1 = \frac{64}{5} - 1 = \frac{59}{5} \quad \therefore 10\alpha^2 = 59 \times 2 = 118$$

27. (7): $\frac{x^2}{16}+\frac{y^2}{9}=1$. Let $(4\cos\theta, 3\sin\theta)$ be the coordinates of the point at which tangent is drawn.

Equation of tangent $\frac{x\cos\theta}{4}+\frac{y\sin\theta}{3}=1$

Let $A = (4\sec\theta, 0)$. $B = (0, 3\,\text{cosec}\,\theta)$

(Tangent intersects coordinate axes at A and B)

$AB^2 = 16\sec^2\theta + 9\,\text{cosec}^2\theta = 25 + 16\tan^2\theta + 9\cot^2\theta$

$AB^2 \ge 25 + 2\times\sqrt{16\tan^2\theta \times 9\cot^2\theta} = 49$; $AB \ge 7$

$\therefore$ $AB_{\min} = 7$

28. (1012) : $\sum_{r=0}^{2023} r^2\, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$

$\sum_{r=0}^{n} r^2 \cdot {}^nC_r = n(n+1)\cdot 2^{n-2}$

Then, $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = (2023)(2023+1)\cdot 2^{2023-2}$

$= 2023 \times 2024 \times 2^{2021} = 2023 \times \alpha \times 2^{2022}$

$\therefore$ $2\alpha = 2024$; $\alpha = 1012$

29. (22): Let $I = 12\int_0^3 |x^2-3x+2|dx = 12\int_0^3 |(x-2)(x-1)|dx$

$$= 12\times\left[\int_0^1 (x^2-3x+2)dx - \int_1^2 (x^2-3x+2)dx + \int_2^3 (x^2-3x+2)dx\right]$$

$$= 12\times\left[\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)_0^1 + \left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)_2^3 - \left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)_1^2\right] = 22$$

30. (60): Even numbers are 2, 4, 2, 4

odd numbers are 1, 3, 1, 3, 1. $\overline{\text{O}}\,\overline{\text{E}}\,\overline{\text{O}}\,\overline{\text{E}}\,\overline{\text{O}}\,\overline{\text{E}}\,\overline{\text{O}}\,\overline{\text{E}}\,\overline{\text{O}}$

Even places can be filled with even numbers in $\frac{4!}{2!\,2!}$ ways

Similarly, odd places can be filled in $\frac{5!}{2!\,3!}$ ways

Required number of ways $= \frac{4!}{2!\,2!} \times \frac{5!}{2!\,3!} = 60$

◆◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(a)	**5.**	(d)
6.	(d)	**7.**	(a,c)	**8.**	(a,d)	**9.**	(a,b,d)	**10.**	(a,c)
11.	(a,c)	**12.**	(b,d)	**13.**	(a,d)	**14.**	(b)	**15.**	(c)
16.	(c)	**17.**	(0.16)	**18.**	(5)	**19.**	(8)	**20.**	(2188)

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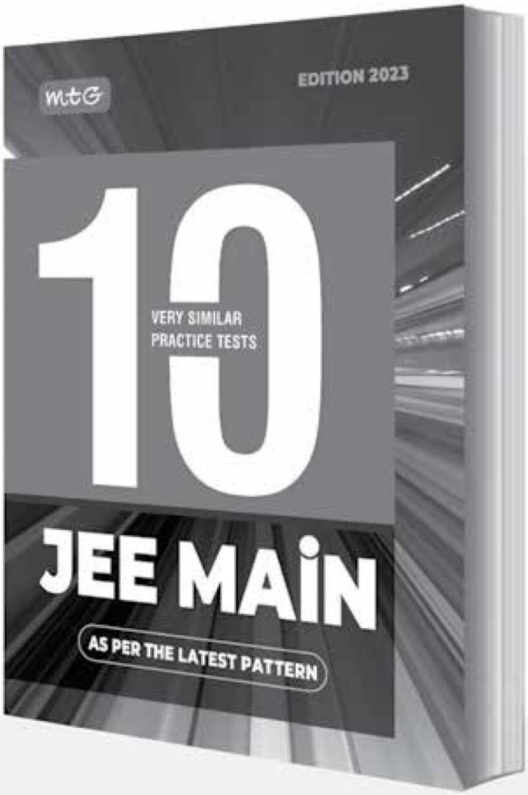
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JEE MAIN

Exam Dates
Session-2
06, 08, 10, 11, 12
April 2023

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The number of different words formed by using the letters 4*A*'s, 2*C*'s, 1*H*, 1*D*, 1*B*, if both *C*'s, do not occur together, are
(a) 7560 (b) 1680 (c) 60 (d) 5880

2. If $y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$,

then $(x^2-1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}$ is equal to
(a) $225y^2$ (b) $224y^2$ (c) $125y$ (d) $225y$

3. The set of all values of λ for which the system of linear equations
$2x_1-2x_2+x_3=\lambda x_1$;
$2x_1-3x_2+2x_3=\lambda x_2$; $-x_1+2x_2=\lambda x_3$
has a non-trivial solution,
(a) contains two elements.
(b) contains more than two elements.
(c) is an empty set.
(d) is a singleton.

4. The probabilities of three events *A*, *B* and *C* are given by $P(A)=0.6$, $P(B)=0.4$ and $P(C)=0.5$. If $P(A\cup B)=0.8$, $P(A\cap C)=0.3$, $P(A\cap B\cap C)=0.2$, $P(B\cap C)=\beta$ and $P(A\cup B\cup C)=\alpha$, where $0.85\le\alpha\le0.95$, then β lies in the interval
(a) [0.35, 0.36] (b) [0.25, 0.35]
(c) [0.20, 0.25] (d) [0.36, 0.40]

5. Let $r\in\{p,q,\sim p,\sim q\}$ be such that the logical statement $r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$ is a tautology. Then r is equal to
(a) p (b) q (c) $\sim p$ (d) $\sim q$

6. The domain of the function

$$f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)\text{ is}$$

(a) $(-2,-1)\cup(-1,0)\cup(2,\infty)$
(b) $(1,2)\cup(2,\infty)$
(c) $(-1,0)\cup(1,2)\cup(2,\infty)$
(d) $(-1,0)\cup(1,2)\cup(3,\infty)$

7. If a variate assumes the values 0, 1, 2, ..., *n*, with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$, then mean square deviation about the value $x=0$ is
(a) $\dfrac{n(n-1)}{n}$ (b) $\dfrac{n^2(n-1)}{4}$
(c) $\dfrac{n(n+1)}{4}$ (d) $\dfrac{n(n+1)}{2}$

8. If $f(x)=\displaystyle\int_{x^2}^{x^3}\frac{dt}{\log t}$, $x>0$ then
(a) $f(x)$ is maximum at $x=1$
(b) $f(x)$ is an increasing function $\forall\ x\in R^+$ only
(c) $f(x)$ is minimum at $x=1$
(d) $f(x)$ is neither maximum nor minimum at $x=1$ but an increasing function $\forall\ x\in R$

9. Let R_1 and R_2 be two relations defined as follows :
$R_1=\{(a,b)\in R^2 : a^2+b^2\in Q\}$ and
$R_2=\{(a,b)\in R^2 : a^2+b^2\notin Q\}$, where *Q* is the set of all rational numbers. Then
(a) Neither R_1 nor R_2 is transitive.
(b) R_2 is transitive but R_1 is not transitive.
(c) R_1 is transitive but R_2 is not transitive.
(d) R_1 and R_2 are both transitive.

10. Let *ABC* be a triangle with $A(-3,1)$ and $\angle ACB=\theta$, $0<\theta<\pi/2$. If the equation of the median through *B* is $2x+y-3=0$ and the equation of angle bisector of *C* is $7x-4y-1=0$, then $\tan\theta$ is equal to
(a) 3/4 (b) 4/3 (c) 2 (d) 1/2

11. If $\displaystyle\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}=\lambda\tan\theta+2\log_e|f(\theta)|+C$,
where *C* is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to
(a) $(-1, 1-\tan\theta)$ (b) $(-1, 1+\tan\theta)$
(c) $(1, 1+\tan\theta)$ (d) $(1, 1-\tan\theta)$

12. If $A=\begin{bmatrix}\cos^2\alpha & \sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha & \sin^2\alpha\end{bmatrix}$;

$B = \begin{bmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{bmatrix}$ are such that, AB is a null matrix, then which of the following should necessarily be an odd integral multiple of $\pi/2$?

(a) α (b) β (c) $\alpha - \beta$ (d) $\alpha + \beta$

13. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is

(a) $\dfrac{2}{\sqrt{21}}$ (b) $2\sqrt{\dfrac{3}{7}}$ (c) $\dfrac{2}{3}\sqrt{\dfrac{7}{3}}$ (d) $\dfrac{2}{3}$

14. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $1/\alpha$ and $1/\beta$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \dfrac{1}{\alpha}\right)\left(\beta - \dfrac{1}{\beta}\right)\left(\alpha + \dfrac{1}{\beta}\right)\left(\beta + \dfrac{1}{\alpha}\right)$ is equal to

(a) $\dfrac{9}{4}(9 + q^2)$ (b) $\dfrac{9}{4}(9 - q^2)$

(c) $\dfrac{9}{4}(9 + p^2)$ (d) $\dfrac{9}{4}(9 - p^2)$

15. The value of $\lim\limits_{x \to 1} \dfrac{(x^2 - 1)\sin^2(\pi x)}{x^4 - 2x^3 + 2x - 1}$ is

(a) $\pi^2/6$ (b) $\pi^2/3$ (c) $\pi^2/2$ (d) π^2

16. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to

(a) 4 (b) 3 (c) 2 (d) 1

17. The integral

$$\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)}\,dx \text{ is equal to}$$

(a) 1 (b) 6 (c) 2 (d) 4

18. Let $z \in C$, the set of complex numbers. Then the equation, $2|z + 3i| - |z - i| = 0$ represents

(a) a circle with radius 8/3.

(b) a circle with diameter 10/3.

(c) an ellipse with length of major axis 16/3.

(d) an ellipse with length of minor axis 16/9.

19. Let $[x]$ denote the greatest integer less than or equal to x. Then $\lim\limits_{x \to 0} \dfrac{\tan(\pi\sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$

(a) equals 0 (b) equals π

(c) equals $\pi + 1$ (d) does not exist

20. Let A be a 3×3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix};\ A\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix} \text{ and } A\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has

(a) no solution

(b) infinitely many solutions

(c) unique solution

(d) exactly two solutions

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. Let $y = y(x)$ be the solution of the differential equation $dy = e^{\mathrm{a}x + y}dx$; $\alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\dfrac{1}{2}\right)$, then the value of α is equal to ________.

22. If b is the first term of an infinite G.P. whose sum is 5, then b lies in the interval (a, b), then $a + b$ is equal to ________.

23. Let the mean and variance of four numbers 3, 7, x and y $(x > y)$ be 5 and 10 respectively. Then the mean of four numbers $3 + 2x$, $7 + 2y$, $x + y$ and $x - y$ is ________.

24. For real numbers α and β, consider the following system of linear equations:

$x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$.

If the system has infinite solutions, then $\alpha + \beta$ is equal to ________.

25. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ________.

26. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then $4A$ is equal to ________.

27. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to ________.

28. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to ________.

29. In a hurdle race, a runner has probability p of jumping over a specific hurdle. Given that in 5 trails, the runner succeeded 3 times, the conditional probability that the runner had succeeded in the first trial is _______.

30. If every solution of the equation $3\cos^2 x - \cos x - 1 = 0$ is a solution of the equation $a\cos^2 2x + b\cos 2x - 1 = 0$. Then the value of $(a + b)$ is equal to _______ .

SOLUTIONS

1. **(d):** Total number of words = Number of words in which particular things are always together + never comes together

$\therefore \quad \frac{9!}{4!\,2!} = \frac{8!}{4!}$ + ways when C's do not come together [Consider $2C$'s as only $1C$, then $n = 8$]

$\therefore$ Number of ways when $2C$'s never come together

$$= \frac{9!}{4!\,2!} - \frac{8!}{4!} = 7560 - 1680 = 5880$$

2. **(d)**

3. **(a):** The system is $(2 - \lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

For non-trivial solution, the determinant of the coefficient matrix must vanish. Then

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \ (2 - \lambda)\{\lambda(3 + \lambda) - 4\} + 2\{-2\lambda + 2\} + 1\{4 - (3 + \lambda)\} = 0$

$\Rightarrow \ (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 4\lambda + 4 + 1 - \lambda = 0$

$\Rightarrow \ (2 - \lambda)(\lambda^2 + 3\lambda - 4) - 5\lambda + 5 = 0$

$\Rightarrow \ (2 - \lambda)(\lambda - 1)(\lambda + 4) - 5(\lambda - 1) = 0$

$\Rightarrow \ (\lambda - 1)(-\lambda^2 - 2\lambda + 8 - 5) = 0$

$\Rightarrow \ (\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0 \Rightarrow (\lambda - 1)^2(\lambda + 3) = 0$

Thus, $\lambda = 1, 1, -3$

$\therefore$ Set of all λ's contain 2 elements.

4. **(b):** We have, $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$.

Consider, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$\Rightarrow \ \alpha = 0.6 + 0.4 + 0.5 - P(A \cap B) - \beta - 0.3 + 0.2$

$\Rightarrow \ \alpha = 1.4 - P(A \cap B) - \beta$ ...(i)

Consider, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \ 0.8 = 0.6 + 0.4 - P(A \cap B)$

$\Rightarrow \ P(A \cap B) = 0.2$ ...(ii)

From (i) and (ii), we get

$\alpha = 1.4 - 0.2 - \beta \Rightarrow \alpha = 1.2 - \beta$

Now, as $0.85 \le \alpha \le 0.95$; $0.85 \le 1.2 - \beta \le 0.95$

$\Rightarrow \ -0.35 \le -\beta \le -0.25 \ \Rightarrow \ 0.25 \le \beta \le 0.35$

5. **(c)**

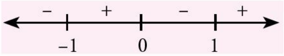
6. **(c):** Given, $f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$

For existence of the given function, $4 - x^2 \ne 0$

$\therefore \ x \in \mathbf{R} - \{-2, 2\}$

Also, $x^3 - x > 0 \Rightarrow \ x(x - 1)(x + 1) > 0$

The sign scheme is



$\therefore \ x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Required domain $= x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

7. **(c):** Mean square deviation about 0 $= \frac{\sum f_i x_i^2}{\sum f_i}$

$$= \frac{\sum\limits_{r=0}^{n} r^2\, {}^nC_r}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n} = \frac{\sum\limits_{r=0}^{n} \left(r(r-1) + r\right) {}^nC_r}{2^n}$$

$$= \frac{1}{2^n}\left[\sum_{r=0}^{n} r(r-1)\, {}^nC_r + \sum_{r=0}^{n} r\, {}^nC_r\right]$$

$$= \frac{1}{2^n}\left[\sum_{r=1}^{n} r(r-1)\frac{n}{r}\, {}^{n-1}C_{r-1} + \sum_{r=1}^{n} r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1}\right]$$

$$= \frac{1}{2^n}\left[\sum_{r=2}^{n} r(r-1)\frac{n}{r} \cdot \frac{n-1}{r-1}\, {}^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\right]$$

$$= \frac{1}{2^n}\left[n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\right]$$

$$= \frac{n(n-1)2^{n-2} + n \cdot 2^{n-1}}{2^n} = \frac{n(n-1)}{4} + \frac{n}{2} = \frac{n(n+1)}{4}$$

8. **(d):** $f(x) = \int\limits_{x^2}^{x^3} \frac{1}{\log t}\,dt$

$\therefore \ f'(x) = \frac{3x^2}{3\log x} - \frac{2x}{2\log x}$ (using Leibnitz theorem)

$\Rightarrow \ f'(x) = \frac{x^2 - x}{\log x} = \frac{x(x-1)}{\log x}$

$\therefore$ For critical points $x(x - 1) = 0 \Rightarrow x = 0, x = 1$

Consider $x = 1$ as $x > 0$

Now, for $x < 1$, $f'(x) = (-\text{ve})\,(-\text{ve}) = \text{positive } (> 0)$

and for $x > 1$, $f'(x) = (+\text{ ve})\ (+\text{ve}) = \text{positive } (> 0)$

$\therefore$ $f'(x)$ does not change its sign in the immediate nbd of $x = 1$. So $x = 1$ is neither the point of maxima nor minima. But $f'(x) > 0\ \forall\ x < 1\ \&\ x > 1$.

$\therefore$ $f(x)$ is an increasing function $\forall\ x \in R$.

9. (a)

10. (b): Let M be the midpoint AC.

Let coordinates of $M \equiv (\alpha, 3 - 2\alpha)$ and $C \equiv \left(\beta, \dfrac{7\beta - 1}{4}\right)$.

$$\text{Now, } \frac{\beta - 3}{2} = \alpha \Rightarrow 2\alpha = \beta - 3 \qquad \text{...(i)}$$

$$\text{and } \frac{7\beta - 1}{4} + 1 = 6 - 4\alpha \Rightarrow 7\beta + 3 = 24 - 16\alpha$$

$$\Rightarrow 7\beta + 3 = 24 - 8 \times (\beta - 3) \qquad \text{[Using (i)]}$$

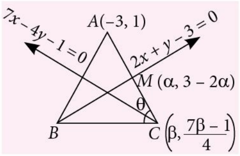
$$\Rightarrow 15\beta = 48 - 3 \Rightarrow 15\beta = 45$$

$$\Rightarrow \beta = 3 \text{ and } \frac{7\beta - 1}{4} = 5$$

$\therefore$ Coordinates of $C \equiv (3, 5)$

Now, slope of $AC = \dfrac{5 - 1}{3 + 3} = \dfrac{2}{3}$

Slope of angle bisector of angle $C = 7/4$


$7x - 4y - 1 = 0$
$A(-3, 1)$
$2x + y - 3 = 0$
$M\ (\alpha, 3 - 2\alpha)$
θ
B
$C\left(\beta, \frac{7\beta - 1}{4}\right)$

$$\text{Now, } \tan\frac{\theta}{2} = \left|\frac{\frac{7}{4} - \frac{2}{3}}{1 + \frac{14}{12}}\right| = \frac{1}{2}$$

$$\therefore \tan\theta = \left|\frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}\right| = \left|\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right| = \frac{4}{3}$$

11. (b): Let $I = \displaystyle\int \frac{\sec^2\theta}{\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right) + \left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)} d\theta$

$$= \int \frac{\sec^2\theta\,(1 - \tan^2\theta)}{(1 + \tan\theta)^2} d\theta = \int \frac{\sec^2\theta\,(1 - \tan\theta)}{1 + \tan\theta} d\theta$$

Putting $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$, we get

$$I = \int \left(\frac{1 - t}{1 + t}\right) dt = \int \left(-1 + \frac{2}{1 + t}\right) dt$$

$= -t + 2\log_e|(1 + t)| + C = -\tan\theta + 2\log_e|(1 + \tan\theta)| + C$

$\therefore$ $\lambda = -1$ and $f(\theta) = 1 + \tan\theta$

12. (c)

13. (a): Given, $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$

and $\vec{a} \cdot \vec{b} = 3$

Projection of $\vec{b}$ on $(\vec{a} - \vec{b})$ is $= \dfrac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}$

Now, $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$$\Rightarrow 5 + 9 = 6|\vec{b}|^2 \Rightarrow \frac{7}{3} = |\vec{b}|^2 \Rightarrow |\vec{b}| = \sqrt{\frac{7}{3}}$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b}) = 6 + \frac{7}{3} - 2 \times 3 = \frac{7}{3}$$

$$\Rightarrow |\vec{a} - \vec{b}| = \sqrt{\frac{7}{3}}$$

Projection $\vec{b}$ on $(\vec{a} - \vec{b})$

$$= \frac{\vec{b} \cdot \vec{a} - |\vec{b}|^2}{|\vec{a} - \vec{b}|} = \frac{3 - \frac{7}{3}}{\sqrt{\frac{7}{3}}} = \frac{2}{3} \times \sqrt{\frac{3}{7}} = \frac{2}{\sqrt{21}}$$

14. (d)

15. (d): $\displaystyle\lim_{x \to 1} \frac{(x^2 - 1)\sin^2 \pi x}{x^4 - 2x^3 + 2x - 1}$

$$= \lim_{x \to 1} \frac{(x + 1)(x - 1)\dfrac{\sin^2(\pi x - \pi)}{\pi^2 (x - 1)^2}\pi^2 (x - 1)^2}{x^4 - 2x^3 + 2x - 1}$$

$$[\because \sin^2(\theta - \pi) = \sin^2\theta]$$

$$= \lim_{x \to 1} \frac{\pi^2 (x + 1)(x - 1)^3}{x^4 - 2x^3 + 2x - 1} \left[\because \lim_{x \to 1}\left(\frac{\sin(\pi(x - 1))}{\pi(x - 1)}\right)^2 = 1\right]$$

$$= 2\pi^2 \lim_{x \to 1} \frac{(x - 1)^3}{x^4 - 2x^3 + 2x - 1} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= 2\pi^2 \lim_{x \to 1} \frac{3(x - 1)^2}{4x^3 - 6x^2 + 2} \qquad \text{[Using L'Hospital's rule]}$$

$$= 2\pi^2 \lim_{x \to 1} \frac{6(x - 1)}{12x^2 - 12x} = 2\pi^2 \lim_{x \to 1} \frac{6(x - 1)}{12x(x - 1)}$$

$$= 2\pi^2 \times \frac{6}{12} = \pi^2$$

16. (b): We have, $y - (x + 2y^2)\dfrac{dy}{dx} = 0$

$$\Rightarrow y = (x + 2y^2)\frac{dy}{dx} \Rightarrow y\frac{dx}{dy} = x + 2y^2$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{1}{y}\right)x = 2y \qquad \text{...(i)}$$

$$\text{I.F.} = e^{\int -\frac{1}{y} dy} = e^{-\ln y} = y^{-1} = \frac{1}{y}$$

$\therefore$ The solution of the equation (i) is

$$x\left(\frac{1}{y}\right) = \int (2y) \times \frac{1}{y} dy + c \Rightarrow \frac{x}{y} = 2y + c \qquad \text{...(ii)}$$

When $x = 1$ and $y = -1$, we get $c = 1$

The equation (ii) becomes $\frac{x}{y} = 2y + 1$ ...(iii)

Put $y = 1$ in (iii), we get $x = 2 + 1 = 3$

17. (a): Let $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(x-6)^2} dx$

$$= \int_2^4 \frac{\log x}{\log x + \log(6-x)} dx \quad (\text{use } \log x^2 = 2\log x,\ x > 0) \qquad \text{...(i)}$$

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, the above rewrite as

$$I = \int_2^4 \frac{\log(6-x)}{\log x + \log(6-x)} dx \qquad \text{...(ii)}$$

Adding (i) & (ii), $2I = \int_2^4 \frac{\log x + \log(6-x)}{\log x + \log(6-x)} dx = \int_2^4 dx = 2$

$\therefore\ I = 1.$

18. (a): We have, $2|z + 3i| - |z - i| = 0$

$\Rightarrow\ 2|x + i(y+3)| = |x + i(y-1)| \qquad (\because z = x + iy)$

$\Rightarrow\ 2\sqrt{x^2 + (y+3)^2} = \sqrt{x^2 + (y-1)^2}$

$\Rightarrow\ 4(x^2 + (y+3)^2) = x^2 + (y-1)^2$

$\Rightarrow\ 3x^2 = y^2 - 2y + 1 - 4y^2 - 24y - 36$

$\Rightarrow\ 3x^2 + 3y^2 + 26y + 35 = 0 \Rightarrow x^2 + y^2 + \frac{26}{3}y + \frac{35}{3} = 0$

This is the equation of circle with radius,

$$r = \sqrt{0^2 + \left(\frac{13}{3}\right)^2 - \frac{35}{3}} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

19. (d): Here,

$$\text{R.H.L.} = \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x) + \left(|x| - \sin(x[x])\right)^2}{x^2}$$

$$= \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x) + x^2}{x^2} \qquad (\because\ x \to 0^+ \Rightarrow [x] = 0)$$

$$= \lim_{x\to 0^+} \frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)} \cdot \frac{\pi\sin^2 x}{x^2} + 1 = \pi + 1$$

Now, $\text{L.H.L.} = \lim_{x\to 0^-} \frac{\tan(\pi\sin^2 x) + (-x + \sin x)^2}{x^2}$

$(\because\ x \to 0^- \Rightarrow [x] = -1)$

$$= \lim_{x\to 0^-} \frac{\tan(\pi\sin^2 x)}{\pi\sin^2 x} \cdot \frac{\pi\sin^2 x}{x^2} + \left(-1 + \frac{\sin x}{x}\right)^2 = \pi$$

$\therefore$ R.H.L. $\neq$ L.H.L.

So, limit does not exist.

20. (b): Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$; $A\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a+b \\ d+e \\ g+h \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$$A\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a+c \\ d+f \\ g+i \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix};\ A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c \\ f \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$c = 1, f = 1, i = 2, a = -2, d = -1, g = -1,$
$b = 3, e = 2, h = 1$

$$A - 2I = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A - 2I)\,X = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4x_1 + 3x_2 + x_3 \\ -x_1 + x_3 \\ -x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow\ -4x_1 + 3x_2 + x_3 = 4$
$-x_1 + x_3 = 1$
$-x_1 + x_2 = 1$

From above equations it can be said that it has infinitely solutions.

21. (2): The given differential equation is,

$$dy = (e^{\alpha x + y})\,dx$$

$$\Rightarrow\ \int \frac{dy}{e^y} = \int e^{\alpha x}\,dx \Rightarrow\ -e^{-y} = \frac{e^{\alpha x}}{\alpha} + c$$

But $y(\log_e 2) = \log_e 2$ [Given]

$$\therefore\ \frac{-1}{e^{\log_e 2}} = \frac{e^{\alpha\log_e 2}}{\alpha} + c \Rightarrow\ c = \frac{-1}{2} - \frac{1}{\alpha}\cdot 2^\alpha \qquad \text{...(i)}$$

Also, $y(0) = \log_e\left(\frac{1}{2}\right)$

$$\Rightarrow\ \frac{-1}{e^{\log_e 1/2}} = \frac{1}{\alpha} + c \Rightarrow c = -2 - \frac{1}{\alpha} \qquad \text{...(ii)}$$

From (i) and (ii), we have $\frac{-1}{2} - \frac{1}{\alpha}\cdot 2^\alpha = -2 - \frac{1}{\alpha}$

$$\Rightarrow\ \frac{1}{\alpha}[1 - 2^\alpha] = -2 + \frac{1}{2} = \left(\frac{1 - 2^2}{2}\right) \Rightarrow\ \alpha = 2$$

22. (10): Let b, br, br^2, ... be an infinite G.P. with first term b, common ratio r and sum = 5.

$\therefore\ 5 = \frac{b}{1-r};\ |r| < 1\ \Rightarrow\ \frac{b}{5} = 1 - r\ \Rightarrow r = 1 - \frac{b}{5}$

Now, $|r| < 1 \Rightarrow \left|1 - \frac{b}{5}\right| < 1\ \Rightarrow\ -1 < 1 - \frac{b}{5} < 1$

$\Rightarrow\ 0 < -\frac{b}{5} + 2 < 2\ \Rightarrow\ 0 < \frac{10 - b}{5} < 2$

$\Rightarrow\ 0 < -b + 10 < 10 \Rightarrow b < 10$ and $b > 0$ or $b \in (0, 10)$

23. (12) : We have, $\dfrac{3+7+x+y}{4}=5$

$\Rightarrow\ x+y=10 \qquad\qquad ...(i)$

and also, we have $\dfrac{3^2+7^2+x^2+y^2}{4}-25=10$

$\therefore\ x^2+y^2=82 \qquad\qquad ...(ii)$

From (i), we have $(x^2+y^2)+2xy=100$

$\Rightarrow\ xy=9 \Rightarrow y=9/x$ [Using (ii)]

Thus, we have $x+\dfrac{9}{x}=10$ [Using (i)]

$\Rightarrow\ x^2-10x+9=0 \Rightarrow (x-9)(x-1)=0 \Rightarrow x=1,9$

Now, $x=1$ implies $y=9$ (rejected as $x>y$)

and $x=9$ implies $y=1$ (accepted)

$\Rightarrow\ 3+2x=21\ ;\ 7+2y=9\ ;\ x+y=10\ ;\ x-y=8$

$\therefore$ Required mean $=\dfrac{21+9+10+8}{4}=12$

24. (5) : $\because$ The system of equations has infinite solutions.

$\therefore$ Det $A=0$, where A is the coefficient matrix.

$$\Rightarrow \begin{vmatrix}1 & 1 & -1\\ 1 & 2 & \alpha\\ 2 & -1 & 1\end{vmatrix}=0$$

$\Rightarrow\ 1(2+\alpha)-(1-2\alpha)-1(-1-4)=0 \Rightarrow \alpha=-2$

Also, $(\text{adj } A)\cdot B=0$, where $B=\begin{bmatrix}2\\ 1\\ \beta\end{bmatrix}$

$$\text{Now, adj } A=\begin{bmatrix}0 & 0 & 0\\ -5 & 3 & 1\\ -5 & 3 & 1\end{bmatrix}$$

$$\therefore\quad (\text{adj } A)\cdot B=\begin{bmatrix}0 & 0 & 0\\ -5 & 3 & 1\\ -5 & 3 & 1\end{bmatrix}\begin{bmatrix}2\\ 1\\ \beta\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}0\\ \beta-7\\ \beta-7\end{bmatrix} \Rightarrow \beta-7=0 \Rightarrow \beta=7$$

$\therefore\ \alpha+\beta=-2+7=5$

25. (10) : The given equation is $z+\alpha|z-1|+2i=0$

Putting $z=x+iy$, we get $x+iy+\alpha|x+iy-1|+2i=0$

$\Rightarrow\ x+\alpha\sqrt{(x-1)^2+y^2}+i(y+2)=0+0i$

$\Rightarrow\ y+2=0$ and $x+\alpha\sqrt{(x-1)^2+y^2}=0$

$\Rightarrow\ y=-2$ and $\alpha^2=\dfrac{x^2}{x^2-2x+5}$

Now, $\dfrac{x^2}{x^2-2x+5}\in\left[0,\dfrac{5}{4}\right]$

PUBLIC NOTICE

JEE Main Session 2

Inviting Online Applications for Joint Entrance Examination (Main) – 2023 Session 2 - Reg.

In continuation of the Public Notice **dated: 15 December 2022**, the National Testing Agency is now inviting online Application Forms for **Joint Entrance Examination (Main) – 2023 Session 2**. The schedule for the same is given below :

Examination / Session	Dates of Examination	Submission of Application Forms Online	Last Date for Payment of Application Fee Online
JEE (Main) – 2023 Session 2	06, 08, 10, 11, and 12 April 2023 (Reserve dates – 13, 15 April 2023)	14 February to 12 March 2023 (up to 09:00 P.M.)	12 March 2023 (up to 11:50 P.M.)

The dates of Advance Intimation of Examination City, downloading of Admit Cards, and declaration of Result will be displayed on JEE (Main) portal in due course.

The candidates who have applied and successfully paid the Examination Fee for JEE (Main) – 2023 Session 1 and wish to appear for JEE (Main) – 2023 Session 2, are required to log in with their previous Application Number and Password as provided in Session 1. They may only choose the **Paper, Medium of the Examination, State code of Eligibility and cities for Session 2 and pay the Examination Fees**.

The candidates are not allowed to fill more than one Application Form. Any candidate with more than one Application Number will be treated as UFM (Unfair Means), even if found at a later stage, and strict action will be taken against that Candidate.

The candidates who have already registered for Session 1 are not required to register again.

Those candidates who have not applied earlier, need to apply afresh as per the schedule given. For details, candidates may refer to the table below.

1.	Existing candidates i.e. candidates registered for Session 1 and Paid Fee be considered for Session 2	**Already registered candidates in Session 1 i.e. existing candidates for Session 2 :** Candidates are allowed to opt for the following: i. *Course (Paper)* ii. *Medium of Question Paper* iii. *State code of Eligibility* iv. *Examination Cities* v. *Fee Payment*
2.	For new candidates, the process for registration and application form filling in Session 2 will remain the same as followed in Session 1.	**For new candidates in Session 2 i.e. not registered including Fee not paid for Session 1 :** Process for registration and Application will remain the same as followed for Session 1.

The Candidates are advised to keep visiting the official websites of NTA **(www.nta.ac.in)** and **(https://jeemain.nta.nic.in/)** for the latest updates.

For further clarification related to JEE (Main) – 2023, the candidates can also contact **011- 40759000/011-69227700** or email at **jeemain@nta.ac.in**.

$$\therefore \quad \alpha^2 \in \left[0, \frac{5}{4}\right] \Rightarrow \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

$$\therefore \quad p = -\frac{\sqrt{5}}{2}; q = \frac{\sqrt{5}}{2}$$

$$\Rightarrow \quad 4(p^2 + q^2) = 4\left(\frac{5}{4} + \frac{5}{4}\right) = 10$$

26. (114) : We have, $f(x) = 2x^3 - 3x^2 - 12x$

$\Rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$

Now, $f'(x) = 0 \Rightarrow x = -1, 2$

and $f''(x) = 12x - 6 \Rightarrow f''(2) > 0 \Rightarrow b = 2$

and $f''(-1) < 0 \Rightarrow a = -1$

Consider, $f(x) = 0 \Rightarrow x(2x^2 - 3x - 12) = 0$

$$\Rightarrow x = 0, \frac{3 \pm \sqrt{105}}{4}$$

Now, $\frac{3 - \sqrt{105}}{4} < -1$ and $\frac{3 + \sqrt{105}}{4} > 2$

The only root lying between –1 & 2 is $x = 0$.

Let, $A_1 = \left|\int_{-1}^{0} (2x^3 - 3x^2 - 12x)\,dx\right|$

$$= \left|\left[\frac{x^4}{2} - x^3 - 6x^2\right]_{-1}^{0}\right| = \left|0 - \left(\frac{1}{2} + 1 - 6\right)\right|$$

$= 6 - \frac{3}{2} = \frac{9}{2}$ sq. units and

$$A_2 = \left|\int_{0}^{2} (2x^3 - 3x^2 - 12x)\,dx\right| = \left|\left[\frac{x^4}{2} - x^3 - 6x^2\right]_{0}^{2}\right|$$

$= |8 - 8 - 24| = 24$

$\therefore \quad A = A_1 + A_2 = 24 + \frac{9}{2} = \frac{57}{2}$ sq. units

$\Rightarrow \quad 4A = 57 \times 2 = 114$ sq. units

27. (9) : We have,

$\vec{a} \cdot \vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1 \Rightarrow \alpha\beta = 2$ ...(i)

$\vec{b} \cdot \vec{c} = 10 \Rightarrow -3\alpha - 2\beta - \alpha = 10$

$\Rightarrow 4\alpha^2 + 2\beta + 10 = 0 \Rightarrow 4\alpha^2 + 4 + 10\alpha = 0$ (Using (i))

$\Rightarrow (\alpha + 2)(4\alpha + 2) = 0$

$\Rightarrow \alpha = -2, \ \alpha = \frac{-1}{2}$ (Not possible, as $\alpha \in Z$)

$\therefore$ From (i), $\beta = -1$

Now, $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k},$

$$\vec{c} = 2\hat{i} - 2\hat{j} + \hat{k} \quad \therefore (\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

$= 1(-1 + 4) - 2(3 - 4) - 1(-6 + 2) = 3 + 2 + 4 = 9$

28. (4) : Let $P \equiv (1, 0, 3)$ and $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

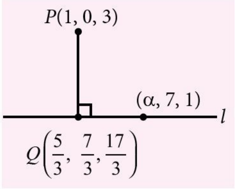
Since, PQ is perpendicular to line l.

$$\therefore \left(1 - \frac{5}{3}\right)\left(\alpha - \frac{5}{3}\right) + \left(\frac{-7}{3}\right)\left(7 - \frac{7}{3}\right)$$

$$+ \left(3 - \frac{17}{3}\right)\left(1 - \frac{17}{3}\right) = 0$$

$$\Rightarrow \frac{-2\alpha}{3} + \frac{10}{9} - \frac{98}{9} + \frac{112}{9} = 0$$

$$\Rightarrow \frac{2\alpha}{3} = \frac{24}{9} \Rightarrow \alpha = 4$$



29. (0.6) : Let A denote the event that the runner succeeds exactly 3 times out of five and B denote the event that the runner succeeds on the first trial.

Required probability $= P(B \mid A) = \frac{P(B \cap A)}{P(A)}$

But $P(B \cap A) = P$ (succeeding in the first trial and exactly twice in other four trials)

$= p({}^4C_2 p^2 (1 - p)^2) = 6p^3(1 - p)^2$

and $P(A) = {}^5C_3 p^3 (1 - p)^2 = 10p^3(1 - p)^2$

Thus, $P(B \mid A) = \frac{6p^3(1 - p)^2}{10p^3(1 - p)^2} = \frac{3}{5}$

30. (13) : From 1st equation, $\cos x = \frac{1 \pm \sqrt{13}}{6}$

Now, $\cos 2x = 2\cos^2 x - 1 = \frac{2}{36}(1 + 13 \pm 2\sqrt{13}) - 1$

$$= \frac{14 \pm 2\sqrt{13} - 18}{18} = \frac{\pm 2\sqrt{13} - 4}{18} = \frac{\pm\sqrt{13} - 2}{9}$$

$\therefore \quad \cos 2x_1 = \frac{\sqrt{13} - 2}{9}$ and $\cos 2x_2 = -\frac{(\sqrt{13} + 2)}{9}$

Now from 2nd equation, $a\cos^2 2x + b\cos 2x - 1 = 0$

$$\therefore \quad \cos 2x_1 \cdot \cos 2x_2 = -\frac{1}{a}$$

$$\Rightarrow \quad -\frac{1}{a} = -\left(\frac{13 - 4}{81}\right) = -\frac{1}{9} \quad \Rightarrow \quad a = 9$$

and $\cos 2x_1 + \cos 2x_2 = -\frac{b}{a} = -\frac{b}{9}$

$$\Rightarrow \quad -\frac{b}{9} = -\frac{4}{9} \quad \Rightarrow \quad b = 4 \ \therefore \ a + b = 13.$$

◆◆

Recipe for Success

"If there is no struggle, there is no progress."

– *Frederick Douglass*

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

Exam on
21st to 31st
May 2023

General Instructions

This practice paper contains two sections i.e. Section A and Section B [B1 and B2].
Section A has 15 questions covering both i.e. Mathematics/Applied Mathematics which is compulsory for all candidates.
Section B1 has 30 questions from Mathematics out of which 20 questions need to be attempted.
Section B2 has 30 questions purely from Applied Mathematics out of which 20 question need to be attempted.

SECTION A

1. If the points (a_1, b_1), (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear, then

(a) $a_1b_2 = a_2b_1$ (b) $a_1 + a_2 = b_1 + b_2$
(c) $a_2b_2 = a_1b_1$ (d) $a_1 + b_1 = a_2 + b_2$

2. Let $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$. Consider the following statements.

Statement-I : $(A + B)^2 = A^2 + 2\ AB + B^2$ for the given matrices.

Statement-II : $AB = BA$ for the above matrices.

In the light of above statement, choose the correct answer from the options given below

(a) Both statement I and statement I are true
(b) Both statement I and statement II are false
(c) Statement I is correct but statement II is false
(d) Statement I is incorrect but statement II is true

3. Match the following columns :

	Column-I	Column-II	
P.	A is a real skew-symmetric matrix such that $A^2 + I = 0$, then AA' is equal to	1.	A
Q.	A is a matrix such that $A^2 = A$ if $(I + A)^n = I + \lambda A$, then	2.	O
R.	If for a matrix A, $A^2 = A$ and $B = I – A$, then $AB + BA + I – (I – A)^2$ equals	3.	$2^n – 1$
S.	If $A' = A$ and $B' = B$, then $(AB – BA)' – (BA – AB)$ equals	4.	I

(a) P-1, Q-2, R-4, S-3 (b) P-2, Q-1, R-3, S-4
(c) P-3, Q-4, R-2, S-1 (d) P-4, Q-3, R-1, S-2

4. The maximum value of the function $f(x) = x^{-x}, x > 0$, is

(a) $1/e$ (b) $e^{1/e}$ (c) e^{-e} (d) e

5. If tangent to the curve $y^2 = x^3$ at its point (m^2, m^3) is also normal to the curve at (M^2, M^3), then what is the value of mM ?

(a) $-4/9$ (b) $-2/9$ (c) $-1/3$ (d) 1

6. If $f'(x) = (x - 1)^3(x - 2)^8$, then

Statement-I : $f(x)$ has neither maximum nor minimum at $x = 2$.

Statement-II : $f'(x)$ changes sign from negative to positive at $x = 2$.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement I are true
(b) Both statement I and statement II are false
(c) Statement I is correct but statement II is false
(d) Statement I is incorrect but statement II is true

7. For the differential equation, $\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{1}{\sqrt{1+x^2}}$,

I. Integrating factor is x

II. $y=\dfrac{\sqrt{1+x^2}}{x}+\dfrac{c}{x}$ III. $y=\dfrac{x}{\sqrt{1+x^2}}+cx$

IV. Integrating factor is $1/x$

(a) All are correct
(b) Only I and II are correct
(c) Only II and III are correct
(d) Only III is correct

8. $\int_0^{1.5}[x^2]\,dx$ is equal to ($[\cdot]$ denotes greatest integer function)

(a) 2 (b) $2-\sqrt{2}$ (c) $2+\sqrt{2}$ (d) $\sqrt{2}$

9. The area bounded between the parabolas $x^2 = \dfrac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$, is

(a) $\frac{20\sqrt{2}}{3}$ sq. units (b) $10\sqrt{2}$ sq. units

(c) $20\sqrt{2}$ sq. units (d) $\frac{10\sqrt{2}}{3}$ sq. units

10. If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = A \cos x + B \log|f(x)| + C$, then:

I. $A = \frac{1}{4}, B = -\frac{1}{\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

II. $A = \frac{1}{2}, B = -\frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

III. $A = -\frac{1}{2}, B = \frac{3}{\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

IV. $A = \frac{1}{2}, B = \frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

V. $A = \frac{-1}{2}, B = \frac{-3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

(a) All are correct
(b) Only I and II are correct
(c) II and IV are correct
(d) Only II and V are correct

11. If X is a random-variable with probability distribution given below:

X	0	1	2	3
$P(X)$	k	$3k$	$3k$	k

The value of k and its variance are

(a) 1/8, 22/27 (b) 1/8, 23/27

(c) 1/8, 24/27 (d) 1/8, 3/4

12. Match the differential equation given in Column-II with their respective solution given in Column-I.

Column-I		Column-II	
P.	$y = ae^{3x} + be^{5x}$	1.	$a + b[(y')^2 + yy''] = 0$
Q.	$ax^2 + by^2 = 1$	2.	$y'' + 16y = 0$
R.	$y = a\sin(4x + b)$	3.	$(y^2 - 2y + 2)y'' + (2y - 2)(y')^2 + e^y y' = 0$
S.	$xe^y - y^2 = 2$	4.	$y'' - 8y' + 15y = 0$

(a) P - 2, Q - 3, R - 4, S - 1
(b) P - 3, Q - 4, R - 1, S - 2
(c) P - 4, Q - 1, R - 2, S - 3
(d) P -1, Q - 2, R - 3, S - 4

13. If the chance that a ship arrives safely at a port is 9/10, find the chance that out of 5 expected ships, atleast 4 will arrive safely at the port.

(a) $\frac{91854}{100000}$ (b) $\frac{32805}{100000}$

(c) $\frac{59049}{100000}$ (d) $\frac{26244}{100000}$

14. In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

(a) $x \geq 6, y \geq 2$
(b) $2x + y \geq 10, x \geq 0, y \geq 0$
(c) $x \geq 6$ (d) none of these

15. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).
Let $F = 4x + 6y$ be the objective function.
The minimum value of F occurs at

(a) (0, 2) only (b) (3, 0) only
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only
(d) any point on the line segment joining the points (0, 2) and (3, 0)

SECTION B1 (MATHEMATICS)

16. Which one of the following relations on R is an equivalence relation?

(a) $aR_1b \Rightarrow |a| = |b|$ (b) $aR_2b \Rightarrow a \geq b$
(c) $aR_3b \Rightarrow a$ divides b (d) $aR_4b \Rightarrow a < b$

17. Solve $\sin^{-1}2x + \cos^{-1}2x + 2\tan^{-1}x = \pi$.

(a) 1 (b) 0
(c) –1 (d) None of these

18. The only integral root of the equation

$$\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix} = 0 \text{ is}$$

(a) $y = 3$ (b) $y = 2$
(c) $y = 1$ (d) none of these

19. If $x + y = \tan^{-1}y$ and $\frac{d^2y}{dx^2} = f(y)\frac{dy}{dx}$, then $f(y) =$

(a) $\frac{-2}{y^3}$ (b) $\frac{2}{y^3}$ (c) $\frac{1}{y}$ (d) $\frac{-1}{y}$

20. Given $0 \leq x \leq \frac{1}{2}$, then the value of

$$\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right\} - \sin^{-1}x\right] \text{ is}$$

(a) 1 (b) $\sqrt{3}$ (c) –1 (d) $1/\sqrt{3}$

21. Match the following :

Column-I		Column-II	
P.	The total number of onto functions from the set {1,2, 3, 4} to the set {3, 4, 7} is	1.	720

Q.	It set A and B contain 4 district elements, then total number of district function from A to B is	2.	5040
R.	If set A has 5 elements and set B has 6 elements, then number of injections that can be formed from A to B is	3.	256
S.	The number of bijective functions from set A to itself, when A contain 7 elements is	4.	36

(a) P – 3, Q – 4, R – 2, S – 1
(b) P – 4, Q – 3, R – 1, S – 2
(c) P – 2, Q – 1, R – 3, S – 4
(d) P – 1, Q – 2, R – 4, S – 3

22. The value of the integral $\int_2^4 \left(\frac{\log t}{t}\right) dt$ is equal to

(a) $\frac{1}{2}(\log 2)^2$ (b) $\frac{5}{2}(\log 2)^2$
(c) $\frac{3}{2}(\log 2)^2$ (d) $(\log 2)^2$

23. The area (in sq. units) bounded by the circle $x^2 + y^2 = 8$, the parabola $x^2 = 2y$ and the line $y = x$ in $y \geq 0$ is

(a) $\frac{2}{3}+2\pi$ (b) $\frac{2}{3}-2\pi$ (c) $\frac{2}{3}+\pi$ (d) $\frac{2}{3}-\pi$

24. The solution of the differential equation
$\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$, where $0 \leq x < \frac{\pi}{2}$,
and $y(0) = 1$, is given by

(a) $y^2=1+\frac{x}{\sec x+\tan x}$ (b) $y=1+\frac{x}{\sec x+\tan x}$
(c) $y=1-\frac{x}{\sec x+\tan x}$ (d) $y^2=1-\frac{x}{\sec x+\tan x}$

25. The equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $1/\sqrt{6}$ from the point $(2, 1, -1)$ is

I. $x + y + z - 3 = 0$
II. $2x - y - z - 3 = 0$
III. $2x - y + z - 3 = 0$
IV. $62x + 29y + 19z - 105 = 0$

(a) All are correct
(b) Only II and IV are correct
(c) Only III and IV are correct
(d) Only I and II are correct

26. Match Column-I with Column-II and select the correct answer by choosing an appropriate option.

Column-I		Column-II	
P.	The slope of the curve $2y^2 = ax^2 + b$ at $(1, -1)$ is -1, then	1.	$a - b = 2$
Q.	If (a, b) be the point on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes, then	2.	$a - b = 7/2$
R.	If the tangent at any point $(1, 2)$ on the curve $y = ax^2 + bx + 7/2$ be parallel to the normal at $(-2, 2)$ on the curve $y = x^2 + 6x + 10$, then	3.	$a - b = 4/3$
		4.	$a+\lvert b\rvert=20/3$
		5.	$5a + 2b = 0$

(a) P-1, Q-3,4, R-2, 5 (b) P-2, Q-3, 4, R-1, 5
(c) P-3, Q-1, 5, R-2, 4 (d) P-4, Q-3, R-1, 2

27. **Statement-I :** A fair coin is tossed fixed number of times. If probability of getting 7 heads is equal to probability of getting 9 heads, then probability of getting 2 heads is equal to $\frac{15}{2^{12}}$.

Statement-II : If ${}^nC_x = {}^nC_y$, then either $x = y$ or $x + y = n$ and $P(X = r) = {}^nC_r\, p^r q^{n-r}$.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement I are true
(b) Both statement I and statement II are false
(c) Statement I is correct but statement II is false
(d) Statement I is incorrect but statement II is true

28. Solve $\frac{dy}{dx} = (4x + y + 1)^2$

(a) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y}{2}\right)=x+c$
(b) $\tan^{-1}\left(\frac{4x+y}{2}\right)=x+c$
(c) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right)=x+c$
(d) $\tan^{-1}\left(\frac{4x+y-1}{2}\right)=x-c$

29. **Statement-I :** The area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is 4/3sq. units.

Statement-II : The area bounded by curves $y = f(x) \geq 0$ and $y = 0$ between ordinates $x = a$ and $x = b$ (where $b > a$) is $\int_a^b f(x)\,dx$.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement I are true

(b) Both statement I and statement II are false

(c) Statement I is correct but statement II is false

(d) Statement I is incorrect but statement II is true

30. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ, then

I. $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2\,|\vec{b}|^2$

II. $|\vec{a}\times\vec{b}|=(\vec{a}\cdot\vec{b})$, if $\theta=\pi/4$

III. $\vec{a}\times\vec{b}=(\vec{a}\cdot\vec{b})\hat{n}$ (where $\hat{n}$ is a normal unit vector), if $\theta = \pi/4$

IV. $(\vec{a}\times\vec{b})\cdot(\vec{a}+\vec{b})=0$

(a) All are correct

(b) Only I and II are correct

(c) II, III and IV are correct

(d) I, II and IV are correct

31. Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 60°$. If $|\vec{a}|=1, |\vec{b}|=2$, then $[(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})]^2$ is equal to

(a) 225 (b) 275 (c) 325 (d) 300

32. If $y = x\tan y$, then $\dfrac{dy}{dx}=$

I. $\dfrac{\tan y}{x-x^2-y^2}$ II. $\dfrac{y}{x-x^2-y^2}$

III. $\dfrac{\tan y}{y-x}$ IV. $\dfrac{\tan x}{x-y^2}$

V. $\dfrac{\tan y}{1-x\sec^2 y}$

(a) All are correct (b) Only I and II are correct

(c) II and V are correct (d) I, II and V are correct

33. The value of $\displaystyle\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ is

(a) $3\sqrt{x}+3(\sqrt[3]{x})-6\sqrt[6]{x}+\log(\sqrt[6]{x}+1)+C$

(b) $2\sqrt{x}+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+C$

(c) $2\sqrt{x}-3(\sqrt[3]{x})+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+C$

(d) None of the above

34. Two tailors P and Q earn ₹350 and ₹450 per day respectively. Tailor P can stitch 6 shirts and 3 trousers while tailor Q can stitch 7 shirts and 3 trousers per day. How many days should each of them work, if it is desired to produce at least 51 shirts and 24 trousers at a minimum labour cost?

(a) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \le 0$, $y \ge 0$

(b) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \ge 0$, $y \ge 0$

(c) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \ge 0$, $y \le 0$

(d) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \le 0$, $y \le 0$

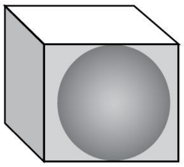
35. If a, b, c are the roots of the equation $x^3-3x^2+3x+7=0$,

then the value of $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$ is

(a) 9 (b) 27 (c) 81 (d) 0

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.



Shreya got a rectangular parallelopiped shaped box and spherical ball inside it as return gift. Sides of the box are x, $2x$, and $x/3$, while radius of the ball is r.

36. If S represents the sum of volume of parallelopiped and sphere, then S can be written as

(a) $\dfrac{4x^3}{3}+\pi r^2$ (b) $\dfrac{2x^2}{3}+\dfrac{4}{3}\pi r^2$

(c) $\dfrac{2x^3}{3}+\dfrac{4}{3}\pi r^3$ (d) $\dfrac{2}{3}x+\dfrac{4}{3}\pi r$

37. If sum of the surface areas of box and ball are given to be constant k^2, then x is equal to

(a) $\sqrt{\dfrac{k^2-4\pi r^2}{6}}$ (b) $\sqrt{\dfrac{k^2-4\pi r}{6}}$

(c) $\sqrt{\dfrac{k^2-4\pi}{6}}$ (d) none of these

38. The radius of the ball, when S is minimum, is

(a) $\sqrt{\dfrac{k^2}{54+\pi}}$ (b) $\sqrt{\dfrac{k^2}{54+4\pi}}$

(c) $\sqrt{\dfrac{k^2}{64+3\pi}}$ (d) $\sqrt{\dfrac{k^2}{4\pi+3}}$

39. Relation between length of the box and radius of the ball can be represented as

(a) $x = 2r$ (b) $x = r/2$ (c) $x = r/3$ (d) $x = 3r$

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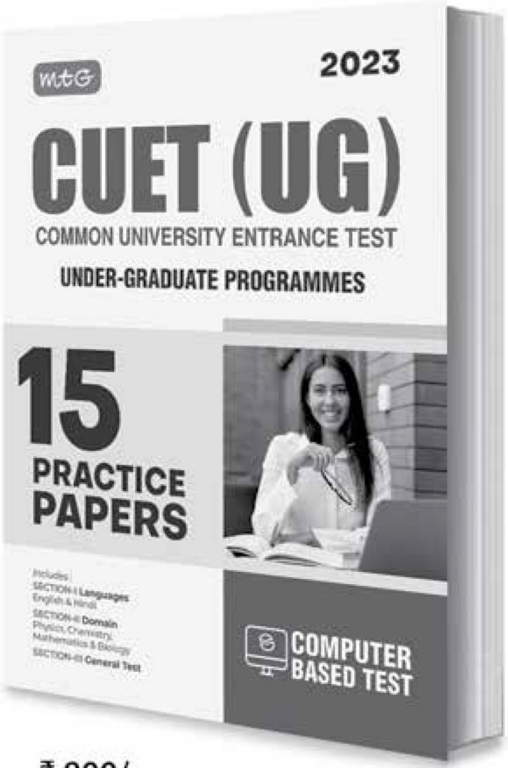

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40. Minimum value of S is

(a) $\dfrac{k^2}{2(3\pi+54)^{2/3}}$ (b) $\dfrac{k}{(3\pi+54)^{3/2}}$

(c) $\dfrac{k^3}{3(4\pi+54)^{1/2}}$ (d) none of these

Case II : Read the following passage and answer the questions from 41 to 45.

A factory has three machines A, B and C to manufacture bolts. Machine A manufacture 30%, machine B manufacture 20% and machine C manufacture 50% of the bolts respectively. Out of their respective outputs 5%, 2% and 4% are defective. A bolt is drawn at random from total production and it is found to be defective.

41. Probability that defective bolt drawn is manufactured by machine A, is

(a) 4/13 (b) 5/13 (c) 6/13 (d) 9/13

42. Probability that defective bolt drawn is manufactured by machine B, is

(a) 0.3 (b) 0.1 (c) 0.2 (d) 0.4

43. Probability that defective bolt drawn is manufactured by machine C, is

(a) $\dfrac{16}{39}$ (b) $\dfrac{17}{39}$ (c) $\dfrac{20}{39}$ (d) $\dfrac{15}{39}$

44. Probability that defective bolt is not manufactured by machine B, is

(a) $\dfrac{35}{39}$ (b) $\dfrac{61}{39}$

(c) $\dfrac{41}{39}$ (d) none of these

45. Probability that defective bolt is not manufactured by machine C, is

(a) 0.03 (b) 0.09 (c) 0.5 (d) 0.9

SECTION B2 (APPLIED MATHEMATICS)

16. The interval in which the function $f(x) = 2x^3 + 3x^2 - 12x + 5$ is strictly increasing is

(a) $[-2, 1]$ (b) $(-\infty, -2] \cup [1, \infty)$

(c) $(-\infty, -2) \cup (0, \infty)$ (d) None of these

17. If $y = \dfrac{\ln x}{x}$, then $\dfrac{d^2y}{dx^2}$ at $(x = 1)$ is equal to

(a) 3 (b) -3 (c) 2 (d) -2

18. Two vessels A and B contain milk and water in the ratio 7 : 5 and 17 : 7 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 5 : 3?

(a) 2 : 1 (b) 1 : 2 (c) 3 : 2 (d) 2 : 3

19. Price relative of rice is 110 in 2019 compared to 2018. If rice costs ₹ 16.50 per kg in 2019, what did it cost in 2018.

(a) ₹ 14 per kg (b) ₹ 14.5 per kg

(c) ₹ 15 per kg (d) ₹ 15.5 per kg

20. Match the following columns:

Column-I		Column-II	
(P)	Average Cost (AC)	(1)	$\dfrac{d}{dx}C(x)$
(Q)	Marginal Cost (MC)	(2)	$\dfrac{1}{x}(MC - AC)$
(R)	Marginal Average Cost (MAC)	(3)	$\dfrac{C(x)}{x}$
(S)	Marginal Revenues	(4)	$\dfrac{d}{dx}R(x)$

(a) P-1, Q-3, R-2, S-4 (b) P-1, Q-3, R-4, S-2

(c) P-3, Q-2, R-1, S-4 (d) P-3, Q-1, R-2, S-4

21. If matrix $A = \begin{bmatrix} a & 5 & c \\ -5 & b & 0 \\ d & 0 & 0 \end{bmatrix}$ is skew symmetric, then the value of $2a + b + c + d$ is equal to

(a) 0 (b) 1 (c) 10 (d) -10

22. If the corner points of the feasible region for an L.P.P. are (0, 3), (5, 0), (6, 0), (2, 3) and (0, 4), then minimum value of objective function $F = 3x + 5y$ occurs at

(a) (0, 3) only

(b) Both (0, 3) and (5, 0)

(c) Mid point of (0, 3) and (5, 0)

(d) Every point of the line segment joining (0, 3) and (5, 0)

23. The probability distribution of a discrete random variable X is given below:

X	20	15	10	5
$P(X)$	1/10	1/5	1/2	1/5

Then $E(X)$ is equal to

(a) 5 (b) 11

(c) 15 (d) None of these

24. 3^{16} mod 4 is equal to

(a) 1 (b) 2 (c) 12 (d) 64

25. In what ratio must a shopkeeper mix two types of oranges worth ₹ 30 per kg and ₹ 45 per kg respectively so as to get a mixture at ₹ 40 per kg?

(a) 3 : 2 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

26. Solve for $x : 3(2 - x) \geq 2(1 - x)$

(a) $x \geq 4$ (b) $x \geq 5$ (c) $x \leq 4$ (d) $x \leq 3$

27. Consider the following statements:

Statement I: The measurable characteristic of sample is called parameter.

Statement II: The measurable characteristic of population is called statistic.

In the light of above statements, choose the correct answer from the options give below.

(a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.

28. If Laspeyre's index number = 196 and Paasche's index number = 64, then Fisher's ideal index number is

(a) 28 (b) 56 (c) 106 (d) 112

29. Which of the following is not a component of time series.

(a) Seasonal (b) Cyclical
(c) Irregular (d) Variance

30. The critical points for the function $f(x) = x^3 - 3x^2 + 3x + 4$ are

(a) 1, 1/3 (b) 1, 3 (c) 1 only (d) 3 only

31. In the given two columns Column II represent the interval of set given in Column I. Match the following columns by choosing appropriate option.

Column I		Column II	
(P)	$\{x : x \in \mathbf{R}, x \leq 2\}$	(1)	$[2, \infty)$
(Q)	$\{y : y \in \mathbf{R}, y \geq 2\}$	(2)	$[-2, 2]$
(R)	$\{z : z \in \mathbf{R}, -2 \leq z \leq 2\}$	(3)	$(-\infty, 2]$
(S)	$\{a : a \in \mathbf{R}, a > 2\}$	(4)	$(2, \infty)$

(a) P-3, Q-1, R-2, S-4 (b) P-1, Q-3, R-2, S-4
(c) P-4, Q-1, R-2, S-3 (d) P-3, Q-4, R-2, S-1

32. Rakesh and Suresh invest in a business in the ratio 4 : 5. If 10% of the profit goes for charity purpose and Rakesh's share of profit is ₹ 18000. Find the total profit.

(a) ₹ 54000 (b) ₹ 45000
(c) ₹ 36000 (d) None of these

33. If the total cost function is given by $(9x - 6x^2 + 5x^3)$, then marginal average cost function (MAC) is given by

(a) $15x^2 - 12x + 9$ (b) $30x - 12$
(c) $-6 + 10x$ (d) None of these

34. A firm anticipates an expenditure of ₹ 500000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of each year into a sinking fund earning interest 5% per annum? (Given: $(1.05)^{10} = 1.629$)

(a) ₹ 39745 (b) ₹ 32435
(c) ₹ 37725 (d) ₹ 42135

35. Consider the following statements:

I. A non singular matrix has a non zero determinant
II. If A is an involuntary matrix then $A^2 = A$
III. A matrix has all elements zero except its diagonal is triangular.
IV. For a orthogonal matrix, $A = A^T$
V. We can not calculate the determinant of a non square matrix.

Which of the above statements is/are true.

(a) I only (b) I, III and V
(c) I, II, IV and V (d) I and V

36. Which of the following is not a type of statistical inferences?

(a) Point Estimation (b) Interval Estimation
(c) Data Estimation (d) Hypothesis Testing

37. Solve the following linear programming problem: Maximize $Z = 60x + 15y$ subject to the constraints $x + y \leq 50$, $3x + y \leq 90$, $x, y \geq 0$. The maximum value of Z is

(a) 1700 (b) 1800 (c) 1750 (d) 1850

38. A dining table costing ₹ 36000 has a useful life of 15 years. If annual depreciation is ₹ 2000, find its scrap value.

(a) ₹ 6000 (b) ₹ 5400 (c) ₹ 4500 (d) ₹ 4200

39. Suppose that $x = t - t^4$, $y = t^2 - t^3$

Statement I: $\dfrac{dy}{dx} = \dfrac{2t - 3t^2}{1 - 4t^3}$

Statement II: $\dfrac{dy}{dx}$ at $(0, 0)$ is $\dfrac{1}{2}$ or 0

(a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.

40. Two pipes A and B together can fill a tank in 10 minutes. If pipe A take 15 minutes less than B to fill the tank alone, then find the time taken by pipe B to fill the tank alone.

(a) 30 min (b) 26 min
(c) 24 min (d) None of these

41. Find the mean, and standard deviation of the probability distribution:

X	2	3	4
$P(X)$	0.2	0.5	0.3

(a) 3.1, 0.49 (b) 3.1, 0.7 (c) 2.9, 0.49 (d) 2.9, 0.7

42. Find the remainder when
672 + 541 + 383 + 295 + 101 + 86 is divided by 3.

(a) 0 (b) 1
(c) 2 (d) Can't determined

43. A boat can row upstream at 10km/h and downstream at 18 km/h. Then

I. Speed of boat in still water is 14 km/h.
II. Speed of boat in still water is 8 km/h.
III. Speed of the stream is 4 km/h.
IV. Speed of the stream is 7 km/h.

Which of the above statement is/are true?

(a) I and III (b) I and IV
(c) II and III (d) II and IV

44. A person wishes to purchase a house for ₹ 5200000 with a down payment of ₹ 700000 and balance in EMI for 25 years. If bank charges 9% per annum compounded monthly, then calculate the EMI.

(Given $(1.0075)^{300} = 9.4084$)

(a) ₹35264 (b) ₹36257
(c) ₹ 37764 (d) ₹ 39241

45. Which of the following is test of adequacy of index numbers?

(a) Time reversal test (b) Factor reversal test
(c) Circular test (d) All of these

SOLUTIONS

SECTION A

1. **(a) :** The given points are collinear.

$$\therefore \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_1+a_2 & b_1+b_2 & 1 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2-a_1 & b_2-b_1 & 0 \\ a_2 & b_2 & 0 \end{vmatrix} = 0$$

Expanding along C_3, we get $b_2(a_2 - a_1) - a_2(b_2 - b_1) = 0$

$\Rightarrow\ -a_1b_2 + a_2b_1 = 0 \Rightarrow a_1b_2 = a_2b_1$

2. **(a) :** Here, $AB = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0+2 & 2+0 \\ 0+2 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

And $BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0+2 & 0+2 \\ 2+0 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Thus, $AB = BA$.

Consequently $(A + B)^2 = A^2 + AB + BA + B^2$
$= A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$.

3. **(d)**

4. **(b) :** $f(x) = x^{-x}, x > 0$

$\Rightarrow\ f(x) = e^{-x\log x} \Rightarrow f'(x) = e^{-x\log x} \times \frac{d}{dx}(-x\log x)$

$\Rightarrow\ f'(x) = x^{-x}(-1 - \log x) = -x^{-x}(1 + \log x)$

For critical points, put $f'(x) = 0$

$\Rightarrow\ -x^{-x}(1 + \log x) = 0 \Rightarrow 1 + \log x = 0$

$[\because x^{-x} \neq 0, \text{ for } x > 0]$

$\Rightarrow\ \log x = -1 \Rightarrow x = e^{-1}$ is the only critical point.

Now, $f''(x) = -x^{-x}\left[\frac{1}{x} - (1 + \log x)^2\right]$

$\therefore\ f''(e^{-1}) = -(e^{-1})^{-e^{-1}}[e - (1-1)^2] = -e^{1/e} \times e = -e^{1+1/e}$

$\Rightarrow\ f''(e^{-1}) < 0$ $[\because e^x > 0 \text{ for all } x > 0]$

So, $x = e^{-1}$ is the point of maxima.

$\therefore$ Maximum value $= (e^{-1})^{-e-1} = e^{1/e}$

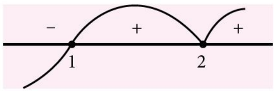
5. **(a) :** We have, $y^2 = x^3 \Rightarrow 2y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$

Given that, slope of tangent at (m^2, m^3)
= slope of normal at (M^2, M^3)

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(m^2,m^3)} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(M^2,M^3)}}$$

$$\Rightarrow \frac{3m^4}{2m^3} = -\frac{2M^3}{3M^4} \Rightarrow mM = -\frac{4}{9}$$

6. **(c) :** It is clear from figure that $f'(x)$ has no sign change at $x = 2$.
Hence, $f(x)$ has neither maximum nor minimum at $x = 2$.



7. **(b) :** Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{\sqrt{1+x^2}}$

It is linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x

Here, I.F. $= e^{\int \frac{1}{x} dx} = x$

∴ Required solution is

$$y \cdot x = \int \frac{x}{\sqrt{1+x^2}} dx + c = \frac{1}{2} \int \frac{2x}{\sqrt{1+x^2}} dx + c$$

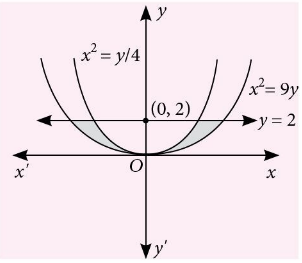
$$\Rightarrow \; y \cdot x = \sqrt{1+x^2} + c \; \Rightarrow \; y = \frac{\sqrt{1+x^2}}{x} + \frac{c}{x}$$

8. (b) : $\int_0^{1.5} [x^2]\, dx = \int_0^1 [x^2]\, dx + \int_1^{\sqrt{2}} [x^2]\, dx + \int_{\sqrt{2}}^{1.5} [x^2]\, dx$

$$= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{1.5} 2 dx = 0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{1.5}$$

$$= \sqrt{2} - 1 + 3 - 2\sqrt{2} = 2 - \sqrt{2}$$

9. (a) : Required area



$$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$$

$$= 2\left[\frac{3y^{3/2}}{3/2} - \frac{1}{2} \cdot \frac{y^{3/2}}{3/2}\right]_0^2$$

$$= 2\left[2(2)^{3/2} - \frac{1}{3}(2)^{3/2} - 0\right]$$

$$= 2\left(4\sqrt{2} - \frac{2\sqrt{2}}{3}\right) = \frac{20}{3}\sqrt{2} \text{ sq. units.}$$

10. (c) : Since the integral is odd w.r.t. sine, we put $\cos x = t$, so $\cos 2x = 2t^2 - 1$ and $dt = -\sin x \, dx$.

$$\therefore \; I = \int \frac{t^2 - 2}{2t^2 - 1} dt = \frac{1}{2} \int \frac{2t^2 - 4}{2t^2 - 1} dt = \frac{1}{2} \int dt - \frac{3}{2} \int \frac{dt}{2t^2 - 1}$$

$$= \frac{1}{2} t - \frac{3}{2\sqrt{2}} \cdot \frac{1}{2} \log \left| \frac{\sqrt{2}t - 1}{\sqrt{2}t + 1} \right| + C$$

$$= \frac{1}{2} \cos x - \frac{3}{4\sqrt{2}} \log \left| \frac{\sqrt{2} \cos x - 1}{\sqrt{2} \cos x + 1} \right| + C$$

So $A = \frac{1}{2}, B = -\frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2} \cos x - 1}{\sqrt{2} \cos x + 1}$

or $A = \frac{1}{2}, B = \frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2} \cos x + 1}{\sqrt{2} \cos x - 1}$

11. (d) : We know, $\Sigma P(X) = 1$

$\Rightarrow \; k + 3k + 3k + k = 1 \; \Rightarrow \; 8k = 1 \Rightarrow k = 1/8$

Now,

X	$P(X)$	$XP(X)$	$X^2P(X)$
0	1/8	0	0
1	3/8	3/8	3/8
2	3/8	6/8	12/8
3	1/8	3/8	9/8
Total		12/8	24/8

Variance $(X) = \Sigma X^2 P(X) - [\Sigma XP(X)]^2$

$$= \frac{24}{8} - \frac{144}{64} = \frac{192 - 144}{64} = \frac{48}{64} = \frac{3}{4}$$

12. (c)

13. (a) : Here, $p = \frac{9}{10}, q = 1 - p = 1 - \frac{9}{10} = \frac{1}{10}$

Probability that atleast 4 will arrive safely at the port = Probability that exactly 4 ships arrive safely + Probability that exactly 5 ships arrive safely

$$= {}^5C_4 \times \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right) + {}^5C_5 \left(\frac{9}{10}\right)^5$$

$$= 5 \times \frac{9^4}{10^5} + \frac{9^5}{10^5} = \frac{14 \times 9^4}{10^5} = \frac{91854}{100000}$$

14. (b) : When $x \ge 6$ and $y \ge 2$, then $2x + y \ge 2 \times 6 + 2$, *i.e.*, $2x + y \ge 14$.

Hence, $x \ge 0$, $y \ge 0$ and $2x + y \ge 10$ are automatically satisfied by every point of the region $\{(x, y) : x \ge 6\} \cap \{(x, y) : y \ge 2\}$

15. (d) : Construct the following table of values of objective function :

Corner Point	Value of $F = 4x + 6y$	
(0, 2)	$4 \times 0 + 6 \times 2 = 12$	} ← minimum
(3, 0)	$4 \times 3 + 6 \times 0 = 12$	
(6, 0)	$4 \times 6 + 6 \times 0 = 24$	
(6, 8)	$4 \times 6 + 6 \times 8 = 72$	← maximum
(0, 5)	$4 \times 0 + 6 \times 5 = 30$	

Since the minimum value $(F) = 12$ occurs at two distinct corner points, it occurs at every point of the segment joining these two points.

SECTION B1 (MATHEMATICS)

16. (a) : Reflexive : $a \in R$, $aR_1 a \Rightarrow |a| = |a|$

∴ R_1 is reflexive.

Symmetric : $a, b \in R$, then $aR_1 b \Rightarrow |a| = |b|$

$\Rightarrow |b| = |a| \Rightarrow bR_1 a$

∴ R_1 is symmetric

Transitive : Let $a, b, c \in R$, then $aR_1 b \Rightarrow |a| = |b|$ ...(i)

and $bR_1 c \Rightarrow |b| = |c|$ ...(ii)

From (i) and (ii), $|a| = |c| \Rightarrow aR_1 c$

∴ R_1 is transitive $\Rightarrow R_1$ is an equivalence relation.

17. (d) : Given trigonometrical equation is

$\sin^{-1} 2x + \cos^{-1} 2x + 2 \tan^{-1} x = \pi$

$$\Rightarrow \; \frac{\pi}{2} + 2\tan^{-1} x = \pi \; \Rightarrow \; 2\tan^{-1} x = \frac{\pi}{2}$$

$\Rightarrow \quad \tan^{-1}x=\frac{\pi}{4} \;\Rightarrow\; x=\tan\left(\frac{\pi}{4}\right)=1$ but $\sin^{-1}2$ is not defined $x = 1$.

18. (c) : Since, $\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix}=0$

$$\Rightarrow \quad (2-y)\,[(5-y)(10-y)-24]-2\,[20-2y-18] + 3\,[8-15+3y]=0$$

$\Rightarrow \quad -y^3+17y^2-43y+27=0$

Only $y = 1$ satisfies this equation.

19. (b) : $x+y=\tan^{-1}y$

$$\Rightarrow \quad 1+\frac{dy}{dx}=\frac{1}{1+y^2}\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=-\frac{1}{y^2}-1$$

$$\frac{d^2y}{dx^2}=-\frac{(-2)}{y^3}\frac{dy}{dx}=\frac{2}{y^3}\frac{dy}{dx}$$

20. (a) : Let $y=\tan\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$

Putting $x = \sin\theta$, we get

$$y=\tan\left[\sin^{-1}\left[\frac{\sin\theta}{\sqrt{2}}+\frac{\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right]-\sin^{-1}(\sin\theta)\right]$$

$$=\tan\left[\sin^{-1}\left[\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right]-\theta\right]$$

$$=\tan\left[\sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right\}-\theta\right]$$

$$=\tan\left[\sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\}-\theta\right]=\tan\frac{\pi}{4}=1$$

21. (b) : (P) Since, we know that if A and B are two sets consisting of m and n elements respectively such that $1 \le n \le m$, then number of onto function from A to B is $\sum_{r=1}^{n}(-1)^{n-r}\,{}^nC_r r^m$ *if* $m \ge n$. Here, $m = 4$, $n = 3$

$\therefore$ Total number of onto functions

$$=\sum_{r=1}^{3}(-1)^{3-r}\,{}^3C_r\,r^4={}^3C_1-{}^3C_2\cdot 2^4+{}^3C_3\cdot 3^4$$

$= 3 - 48 + 81 = 36$

(Q) We know that if A and B are two sets consisting m and n elements respectively, then total number of functions from A to B is n^m. Here, $n = m = 4$

$\therefore$ Total number of functions $= 4^4 = 256$

(R) We know that if A and B are two sets consisting m and n elements respectively then total number of injections that can be formed are ${}^nP_m$, $n \ge m$.

Here, $m = 5$ and $n = 6$

$\therefore$ Total number of one one functions from A to $B={}^6P_5=\frac{6!}{(6-5)!}=\frac{6\times5\times4\times3\times2\times1}{1!}=720$

(S) We know that if A and B are two sets consisting m and n elements respectively, then number of bijections that can be formed are $m!$, if $m = n$.

$\therefore$ Total number of bijective functions from set A to itself $= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

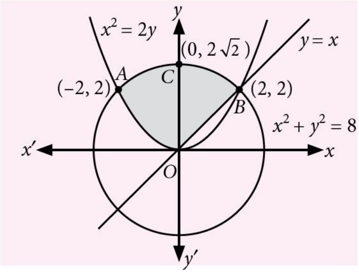
22. (c) : Let $\log t = x \Rightarrow \frac{1}{t}dt=dx$

$$\int_{\log 2}^{\log 4} x\,dx=\left[\frac{x^2}{2}\right]_{\log 2}^{\log 4}=\frac{1}{2}\left[(\log 2^2)^2-(\log 2)^2\right]$$

$$=\frac{1}{2}\left[(\log 2^2-\log 2)(\log 2^2+\log 2)\right]$$

$$=\frac{1}{2}\left[(\log 2)\,(3\log 2)\right]=\frac{3}{2}(\log 2)^2$$

23. (a) : We have, $x^2 + y^2 = 8$, $x^2 = 2y$ and $y = x$



Required area = area of shaded region $AOBC$

$$=\int_{-2}^{2}\sqrt{8-x^2}\,dx-\int_{-2}^{0}\frac{x^2}{2}dx-\int_{0}^{2}x\,dx$$

$$=2\int_{0}^{2}\sqrt{8-x^2}\,dx-\left[\frac{x^3}{2\times3}\right]_{-2}^{0}-\left[\frac{x^2}{2}\right]_{0}^{2}$$

$\left[\because \sqrt{8-x^2}\text{ is an even function}\right]$

$$=2\left[\frac{x}{2}\sqrt{8-x^2}+\frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2-\frac{1}{6}(0+8)-\frac{1}{2}(4-0)$$

$$=2\left[\frac{2}{2}(2)+4\left(\frac{\pi}{4}\right)-0-0\right]-\frac{4}{3}-2=\left(2\pi+\frac{2}{3}\right)\text{sq. units}$$

24. (d) : We have, $\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}$

$$\Rightarrow \quad 2y\frac{dy}{dx}+y^2\sec x=\tan x \qquad \ldots\text{(i)}$$

Put $y^2=t\Rightarrow 2y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore$ (i) becomes, $\frac{dt}{dx}+t\sec x=\tan x$

$$\text{I.F.}=e^{\int\sec x\,dx}=e^{\log(\sec x+\tan x)}=\sec x+\tan x$$

$\therefore$ Solution is given by

$$t(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx$$

$$=\int \tan x\sec x dx+\int \tan^2 x dx$$

$$\Rightarrow \quad t(\sec x + \tan x) = \sec x + \tan x - x + c$$

$$\Rightarrow \quad t=1-\left(\frac{x-c}{\sec x+\tan x}\right) \Rightarrow y^2=1-\left(\frac{x-c}{\sec x+\tan x}\right)$$

Now, $y(0) = 1 \Rightarrow 1 = 1 - (0 - c) \Rightarrow c = 0$

$\therefore$ Particular solution is $y^2=1-\dfrac{x}{\sec x+\tan x}$

25. (c) : The plane is

$2x - y + z - 3 + \lambda(3x + y + z - 5) = 0$ ...(i)

$\Rightarrow \quad (2 + 3\lambda)x + (\lambda - 1)y + (\lambda + 1)z - (5\lambda + 3) = 0$

Its distance from (2, 1, –1) is $\dfrac{1}{\sqrt{6}}$. [Given]

$$\therefore \quad \frac{|4+6\lambda+\lambda-1-\lambda-1-5\lambda-3|}{\sqrt{(2+3\lambda)^2+(\lambda-1)^2+(\lambda+1)^2}}=\frac{1}{\sqrt{6}}$$

$$\Rightarrow 6(\lambda - 1)^2 = 11\lambda^2 + 12\lambda + 6$$

$$\Rightarrow \quad (5\lambda + 24)\lambda = 0 \Rightarrow \lambda=0, \lambda=\frac{-24}{5}$$

For $\lambda = 0,\ 2x - y + z - 3 = 0$

For $\lambda=\dfrac{-24}{5}$, $5(2x - y + z - 3) - 24(3x + y + z - 5) = 0$

$\Rightarrow \quad 62x + 29y + 19z - 105 = 0.$

26. (a) : (P) Here, $2y^2 = ax^2 + b$

$$\Rightarrow \quad 4y\frac{dy}{dx}=2ax \Rightarrow \left[\frac{dy}{dx}\right]_{(1,-1)}=\frac{a}{-2}=-1 \qquad \text{[Given]}$$

$\Rightarrow \quad a = 2$

Also, (1, –1) lies on $2y^2 = ax^2 + b$

$\Rightarrow \quad 2 = a + b \Rightarrow b = 0 \Rightarrow a - b = 2$

(Q) $\because (a, b)$ lies on the curve $9y^2 = x^3$

$\therefore \quad 9b^2 = a^3$ ...(i)

Now, $18y\dfrac{dy}{dx}=3x^2$ and $\dfrac{dy}{dx}=\dfrac{x^2}{6y} \Rightarrow \left[\dfrac{dy}{dx}\right]_{(a,b)}=\dfrac{a^2}{6b}$

Since, the normal to the curve at (a, b) makes equal intercepts with the coordinate axes. Therefore, slope of normal = ± 1.

$$\Rightarrow \quad -\frac{1}{\left[\frac{dy}{dx}\right]_{(a,b)}}=\pm 1 \quad \Rightarrow \left[\frac{dy}{dx}\right]_{(a,b)}=\pm 1 \Rightarrow \frac{a^2}{6b}=\pm 1$$

$$\Rightarrow \quad a^2 = \pm 6b \quad \Rightarrow a^4 = 36b^2 = 36\left(\frac{a^3}{9}\right) \qquad \text{[From (i)]}$$

$\therefore \quad a = 0, 4 \Rightarrow b = 0, \pm 8/3.$

But the line making equal intercepts with the coordinate axes can not pass through the origin.

Hence, the required points are (4, 8/3) and (4, –8/3)

$\therefore a - b = 4/3$ and $a - b = 20/3$ and $a + |b| = 4 + 8/3 = 20/3$

(R) We have, $y = ax^2 + bx + 7/2 \Rightarrow \dfrac{dy}{dx} = 2ax + b$

$\because \quad (1, 2)$ lies on $y = ax^2 + bx + 7/2$

$\therefore \quad 2 = a + b + 7/2 \Rightarrow a + b + 3/2 = 0$ ....(i)

$\left[\dfrac{dy}{dx}\right]_{(1,2)} = 2a(1) + b = 2a + b$ ...(ii)

Also, we have $y = x^2 + 6x + 10 \Rightarrow \dfrac{dy}{dx} = 2x + 6$

$$\therefore \quad -\frac{1}{\left[\frac{dy}{dx}\right]_{(-2,2)}}=-\frac{1}{2(-2)+6}=-\frac{1}{2} \qquad \text{....(iii)}$$

Given, $2a + b = -1/2$ ...(iv)

Solving (i) and (iv), we get $a = 1, b = -5/2$

$\Rightarrow a - b = 7/2$ and $5a + 2b = 0$

27. (d) : Let n be the number of times a fair coin is tossed and X be the number of times head occurs.

$\therefore \quad p=\dfrac{1}{2}=q$, Now, $P(7) = P(9)$

$$\Rightarrow {}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}={}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$$\Rightarrow \quad {}^nC_7 = {}^nC_9 \Rightarrow n = 16$$

$$\therefore \quad P(2)={}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14}=\frac{15}{2^{13}}$$

$\therefore$ Statement-I is false but Statement-II is true.

28. (c) : Putting $4x + y + 1 = t \Rightarrow 4+\dfrac{dy}{dx}=\dfrac{dt}{dx}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{dt}{dx}-4$

Given equation becomes $\dfrac{dt}{dx}-4=t^2 \Rightarrow \dfrac{dt}{t^2+4}=dx$

Integrating both sides, $\displaystyle\int\frac{dt}{4+t^2}=\int dx$

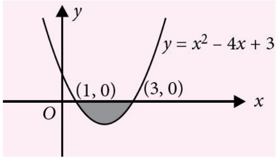
$$\Rightarrow \quad \frac{1}{2}\tan^{-1}\frac{t}{2}=x+c \Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right)=x+c$$

29. (a) : We have, $y = x^2 - 4x + 3$

$\therefore$ Required area $=\displaystyle\int_1^3 -(x^2-4x+3)dx$

$$=-\left[\frac{x^3}{3}-\frac{4x^2}{2}+3x\right]_1^3$$

$$=\frac{4}{3}\text{sq.units}$$



$\therefore$ Statement-I is true.

Obviously, Statement-II is true.

30. (a)

31. (d): We have, $(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})$

$$=3(\vec{a}\times\vec{a})-(\vec{a}\times\vec{b})+9(\vec{b}\times\vec{a})-3(\vec{b}\times\vec{b})$$

$$=-(\vec{a}\times\vec{b})-9(\vec{a}\times\vec{b}) \quad [\because \vec{a}\times\vec{a}=0 \text{ and } -(\vec{a}\times\vec{b})=\vec{b}\times\vec{a}]$$

$$=-10(\vec{a}\times\vec{b})$$

$$\therefore\ [(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})]^2=100(\vec{a}\times\vec{b})^2$$

Now, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin 60°=1\times 2\times\frac{\sqrt{3}}{2}=\sqrt{3}$

$$\therefore\ 100(\vec{a}\times\vec{b})^2=100\times(\sqrt{3})^2=300$$

32. (c)

33. (c) : Put $I=\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$;

Now, put $x=t^6\Rightarrow dx=6t^5\,dt$

$$\Rightarrow I=\int\frac{6t^5}{t^3+t^2}\,dt=6\int\frac{t^3}{t+1}\,dt=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$$

$$=2t^3-3t^2+6t-6\log(t+1)+C$$

$$=2\sqrt{x}-3(\sqrt[3]{x})+6(\sqrt[6]{x})-6\log(\sqrt[6]{x}+1)+C$$

34. (b): Let P and Q work for x and y days respectively.

$\therefore$ Total labour cost $=350x+450y$

	P	**Q**	**Minimum requirements**
Shirts	6	7	51
Trousers	3	3	24

Constraints are $6x+7y\geq 51,\ 3x+3y\geq 24,\ x\geq 0,\ y\geq 0$

$\therefore$ Given problem can be formulated as

Minimize $Z=350x+450y$

Subject to, $6x+7y\geq 51,\ 3x+3y\geq 24,\ x\geq 0,\ y\geq 0$

35. (d)

36. (c) : Let S be the sum of volume of parallelopiped and sphere, then

$$S=x(2x)\left(\frac{x}{3}\right)+\frac{4}{3}\pi r^3=\frac{2x^3}{3}+\frac{4}{3}\pi r^3$$

37. (a) : Since, sum of surface area of box and sphere is given to be constant k^2.

$$\therefore\ 2\left(x\times 2x+2x\times\frac{x}{3}+\frac{x}{3}\times x\right)+4\pi r^2=k^2$$

$$\Rightarrow\ 6x^2+4\pi r^2=k^2$$

$$\Rightarrow\ x^2=\frac{k^2-4\pi r^2}{6}\ \Rightarrow\ x=\sqrt{\frac{k^2-4\pi r^2}{6}}$$

38. (b)

39. (d): Since, $x^2=\frac{k^2-4\pi r^2}{6}=\frac{1}{6}\left[k^2-4\pi\left(\frac{k^2}{54+4\pi}\right)\right]$

$$=\frac{9k^2}{54+4\pi}=9\left(\frac{k^2}{54+4\pi}\right)=9r^2=(3r)^2$$

$\Rightarrow x=3r$

40. (c) : Minimum value of S is given by

$$\frac{2}{3}(3r)^3+\frac{4}{3}\pi r^3=18r^3+\frac{4}{3}\pi r^3=\left(18+\frac{4}{3}\pi\right)r^3$$

$$=\left(18+\frac{4}{3}\pi\right)\left(\frac{k^2}{54+4\pi}\right)^{3/2}=\frac{1}{3}\frac{k^3}{(54+4\pi)^{1/2}}$$

(41-45) : Let E_1, E_2, E_3 be the events of drawing a bolt produced by machine A, B and C, respectively.

Then, $P(E_1)=\frac{30}{100}=\frac{3}{10}, P(E_2)=\frac{20}{100}=\frac{1}{5}$

and $P(E_3)=\frac{50}{100}=\frac{1}{2}$

Also, let E be the event of drawing a defective bolt.

Then, $P(E|E_1)=\frac{5}{100}=\frac{1}{20}, P(E|E_2)=\frac{2}{100}=\frac{1}{50}$,

$P(E|E_3)=\frac{4}{100}=\frac{1}{25}$

41. (b)

42. (b): Probability that the defective bolt is manufactured by machine $B=P(E_2|E)$

$$=\frac{P(E|E_2)\cdot P(E_2)}{P(E|E_1)\cdot P(E_1)+P(E|E_2)\cdot P(E_2)+P(E|E_3)\cdot P(E_3)}$$

$$=\frac{\frac{1}{50}\times\frac{1}{5}}{\left(\frac{1}{20}\times\frac{3}{10}\right)+\left(\frac{1}{50}\times\frac{1}{5}\right)+\left(\frac{1}{25}\times\frac{1}{2}\right)}=\frac{1}{250}\times\frac{1000}{39}=\frac{4}{39}\approx 0.1$$

43. (c)

44. (a) : Probability that the defective bolt is not manufactured by machine B *i.e.*, it is manufactured by machine A or C $=\frac{15}{39}+\frac{20}{39}=\frac{35}{39}$.

45. (c) : Required probability $=\frac{15}{39}+\frac{4}{39}=\frac{19}{39}\approx 0.5$.

SECTION B2 (APPLIED MATHEMATICS)

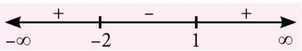
16. (b): $f(x)=2x^3+3x^2-12x+5; f'(x)=6x^2+6x-12$

$=6(x^2+x-2)=6(x+2)(x-1)$

For critical point $f'(x)=0$

$\Rightarrow\ 6(x+2)(x-1)=0$

$\Rightarrow\ x=-2$ and $x=1$



Now, check the value of function $f(x)$ for different values of x, we get

Function $f'(x)$ is negative between $x=-2$ and 1

$\therefore\ f(x)$ is increasing in $(-\infty,-2]\cup[1,\infty)$

17. (b) **18. (a)**

19. (c) : When index number is 110 then cost of rice is ₹ 16.50 per kg.

So, when index number is 1, then cost of rice will be ₹ $\frac{16.50}{110}$. per kg

When index number is 100, then cost of rice will be ₹ $\frac{16.50}{110}\times 100$ per kg *i.e.*, 15 per kg

i.e., cost of rice in 2018 will be ₹ 15 per kg

[$\because$ We assume that index number of base year is 100]

20. (d): Average Cost (AC) $=\frac{C(x)}{x}$

Marginal Cost $=\frac{d}{dx}C(x)$

Marginal Average Cost $=\frac{1}{x}(MC-AC)$

Marginal Revenue $=\frac{d}{dx}R(x)$

21. (a): $A=\begin{bmatrix} a & 5 & c \\ -5 & b & 0 \\ d & 0 & 0 \end{bmatrix}$ is skew symmetric

$\therefore$ Diagonal element must be zero $\Rightarrow a = b = 0$

Also $a_{ij} = -a_{ji}$ for $1 \le i \le 3$ and $1 \le j \le 3$. $\Rightarrow a_{31} = -a_{13}$

$\Rightarrow d = -c$

Now, $2a + b + c + d = c + d$ $[\because a = b = 0]$

$= c - c$ $[\because d = -c]$

$= 0$

22. (d): The value of function $F = 3x + 5y$ is minimum at (0, 3) and (5, 0)

So, every point on the line segment joining (0, 3) and (5, 0) will give minimum value.

23. (b): $E(x)=\sum_i x_i p(x_i)$

$= 20\times\frac{1}{10}+15\times\frac{1}{5}+10\times\frac{1}{2}+5\times\frac{1}{5} = 2 + 3 + 5 + 1 = 11$

24. (a) : $3^2 = 9 = 1 \bmod 4$

$3^{16} = (3^2)^8 = 1^8 \bmod 4 \Rightarrow 3^{16} \equiv 1 \pmod 4$

$\therefore$ 39 (mod 4) $\equiv$ 1.

25. (b): Here,

The C.P. of cheaper oranges '*c*' = ₹30 per kg,

The C.P. of dearer oranges '*d*' = ₹45 per kg

and the mean price of the mixture '*m*' = ₹40 per kg

So, $\frac{\text{quantity of cheaper oranges}}{\text{quantity of dearer oranges}}=\frac{d-m}{m-c}$

$=\frac{45-40}{40-30}=\frac{5}{10}=\frac{1}{2}$.

Hence, the required ratio is 1 : 2.

PUBLIC NOTICE

CUET 2023

National Testing Agency has been mandated by the **Ministry of Education and UGC** to conduct Common University Entrance Test (CUET) for admission into **Undergraduate Programmes** in **Central Universities under Ministry of Education, and other participating Universities / Institutions / Organizations / Autonomous Colleges**.

CUET (UG) will provide a **single window opportunity to students seeking admission in any of the Central Universities (CUs) or other participating organizations (including State Universities, Deemed and Private Universities) across the Country**.

For the Academic Session **2023-24, CUET (UG) – 2023** will be conducted in the **Computer Based Test (CBT) mode** as per the details given below :

Online Submission of Application Form	**09 February to 12 March 2023 (Up to 09:00 P.M.)**
Last date of successful transaction of fee through Credit / Debit Card / Net-Banking / UPI	**12 March 2023 (up to 11:50 P.M.)**
Correction in Particulars	**15 March to 18 March 2023 (Up to 11:50 P.M.)**
Announcement of the City of Examination	**30 April 2023**
Downloading of Admit Cards from the NTA website	**Second week of the May 2023**
Date of Examination	**21 May 2023 onwards**
Display of Recorded Responses and Answer Keys	To be announced later on the website
Website(s)	www.nta.ac.in, https://cuet.samarth.ac.in/
Declaration of Result on the NTA website	*To be announced later on the website*

The details of the programmes offered by Central Universities (CUs) and other participating Universities / Organizations are available on their respective websites as well as on https://cuet.samarth.ac.in/ **Candidates must carefully refer to the eligibility requirements of the programmes offered by various Central Universities and participating Universities / Organizations in this regard**.

The CUET (UG) - 2023 will be conducted in **13 languages** i.e. **English, Hindi, Assamese, Bengali, Gujarati, Kannada, Malayalam, Marathi, Odia, Punjabi, Tamil, Telugu, and Urdu**.

For CUET (UG) - 2023, the following is the scheme of examination :

Section	Subjects/ Tests	Questions to be Attempted
Section 1A - Languages	There are 13 different languages. Any of these languages may be chosen.	40 questions to be attempted out of 50 in each language
Section 1B - Languages	There are 20 Languages. Any of these languages may be chosen.	
Section 2 - Domain Specific Subject	There are 27 Domain specific subjects being offered under this section. A candidate may choose any subject as desired by the applicable University/ Organization.	35/40 Questions to be attempted out of 45/50
Section 3 - General Test	For any such undergraduate programme / programmes being offered by Universities where a General Test is being used for admission.	50 Questions to be attempted out of 60

Note:

1. *From the above subjects / languages, the* ***candidate can choose maximum of 10 subjects from all three Section.***
2. ***Examination will be conducted on multiple days in three shifts, depending on the number of Candidates and Subject choices.***

In case any candidate faces difficulty in applying for **CUET (UG) – 2023**, he/she may contact **011 - 40759000 / 011 - 69227700** or e-mail at **cuet-ug@nta.ac.in**. The candidates are also advised to be in touch with the NTA website(s) www.nta.ac.in and https://cuet.samarth.ac.in/ for the latest updates regarding the exam.

26. (c) : Given $3(2-x) \geq 2(1-x) \Rightarrow 6-3x \geq 2-2x$
$\Rightarrow \quad 6-3x+2x \geq 2 \Rightarrow 6-x \geq 2 \Rightarrow -x \geq 2-6$
$\Rightarrow -x \geq -4 \Rightarrow x \leq 4$

27. (b) : Both statements are incorrect as the measurable characteristic of population is called parameter and measurable characteristic of sample is called statistic.

28. (d) : Fisher's ideal index number

$= \sqrt{\text{Laspeyre's Index} \times \text{Paache's Index}} = \sqrt{196 \times 64}$

$= 14 \times 8 = 112$

29. (d) : Variance is not a time series component, it refers to the spread of a data set.

30. (c) : $f(x) = x^3 - 3x^2 + 3x + 4$; $f'(x) = 3x^2 - 6x + 3$.
For critical point, put $f'(x) = 0$ *i.e.*, $3x^2 - 6x + 3 = 0$
$\Rightarrow \quad 3(x^2 - 2x + 1) = 0 \Rightarrow (x-1)^2 = 0$
$\Rightarrow \quad x = 1$ is the critical point

31. (a)

32. (b) : Let the total profit be ₹x.
$\because$ 10% of profit goes in charity,
$\therefore$ Profit shares by both the partner = 90% of $x = \frac{90x}{100} = \frac{9x}{10}$.

Now, Rakesh share of profit $= \frac{4}{9} \times \frac{9}{10} x \Rightarrow 18000 = \frac{2}{5} x$

$\Rightarrow x = 45000$. Hence, the total profit is ₹45000.

33. (c) : $C(x) = 9x - 6x^2 + 5x^3$

$$\text{Average cost (AC)} = \frac{\text{Total Cost}}{\text{Output}} = \frac{C(x)}{x} = 9 - 6x + 5x^2$$

$$\text{Marginal average cost} = \frac{d}{dx}(AC) = -6 + 10x.$$

34. (a)

35. (d) : Only I and V are true statement.

36. (c) : Data estimation is not a type of statistical inference.

37. (b)

38. (a) : Given original cost of dining table = ₹36000
Useful life = 15 years, Annual depreciation = ₹2000
Let the scarp value of the dining table be ₹S, the using

$$\text{annual depreciation} = \frac{\text{original cost} - \text{scrap value}}{\text{useful life}}$$

we, get $2000 = \frac{36000 - S}{15} \Rightarrow 30000 = 36000 - S \Rightarrow S = 6000$

Hence, the scrap value of dining table is ₹6000.

39. (c) : $\frac{dx}{dt} = 1 - 4t^3$ and $\frac{dy}{dt} = 2t - 3t^2$

$$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t - 3t^2}{1 - 4t^3}$$

Now, at $x = 0, y = 0$ we get, $t - t^4 = 0$ and $t^2 - t^3 = 0$
$t(1 - t^3) = 0$ and $t^2(1 - t) = 0 \Rightarrow t = 0$ and 1 is common value.

$$\therefore \quad \left.\frac{dy}{dx}\right|_{t=0} = 0 \text{ and } \left.\frac{dy}{dx}\right|_{t=1} = \frac{2-3}{1-4} = \frac{-1}{-3} = \frac{1}{3}$$

$$\therefore \quad \frac{dy}{dx} \text{ at } (0, 0) \text{ is } 0 \text{ or } \frac{1}{3}.$$

40. (a)

41. (b) : We construct the following table :

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
Total		3.1	10.1

$\therefore \quad$ Mean $= \mu = 3.1$

$$\text{Variance} = \sum p_i x_i^2 - \mu^2 = 10.1 - (3.1)^2 = 10.1 - 9.61 = 0.49$$

Standard deviation $= \sqrt{0.49} = 0.7$

42. (c) : We find $(672 + 541 + 383 + 295 + 101 + 86) \bmod 3$
Since $672 \equiv 0 \pmod 3$, $541 \equiv 1 \pmod 3$, $383 \equiv 2 \pmod 3$, $295 \equiv 1 \pmod 3$, $101 \equiv 2 \pmod 3$ and $86 \equiv 2 \pmod 3$.
So, $(672 + 541 + 383 + 295 + 101 + 86) \bmod 3$
$\equiv (0 + 1 + 2 + 1 + 2 + 2) \bmod 3 \equiv 8 \pmod 3 \equiv 2 \pmod 3$.
Hence, the remainder when $(672 + 541 + 383 + 295 + 101 + 86)$ is divided by 3 is 2.

43. (a) : Given, upstream speed $v = 10$ km/h
and downstream speed $u = 18$ km/h
then speed of boat in still water

$x = \frac{u+v}{2} = \frac{18+10}{2} = 14$ km/h and speed of stream

$y = \frac{u-v}{2}$ and $\frac{18-10}{2} = 4$ km/h.

44. (c) : Cost of house = ₹ 5200000
Down payment = ₹ 700000 $\therefore$ Balance = ₹ 4500000

So, $P =$ ₹ 4500000, $i = \frac{9}{12 \times 100} = 0.0075$
and $n = 25 \times 12 = 300$

$$\therefore \quad \text{EMI} = \frac{4500000 \times 0.0075 \times (1.0075)^{300}}{(1.0075)^{300} - 1}$$

$$= \frac{33750 \times 9.4084}{9.4084} = \frac{33750 \times 9.4084}{8.4084} = 37763.84$$

Hence, EMI = ₹ 37764

45. (d) : All of the given tests are the test of adequacy of index numbers.

Exam on 30th April 2023

WB JEE

PRACTICE PAPER 2023

Category-I (Q.1 to 50)

(Carry 1 mark each. Only one option is correct. Negative marks : ¼)

1. For real numbers x and y, we write $xRy \Leftrightarrow x-y+\sqrt{2}$ is an irrational number. Then, the relation R is

(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

2. 28 is divided into 4 parts which are in A.P. The ratio of the product of the first and third to the product of the second and fourth is 8 : 15. The largest part is

(a) 6 (b) 8 (c) 10 (d) 12

3. The summation of two unit vectors is a third unit vector, then the modulus of the difference of these unit vectors is

(a) $\sqrt{3}$ (b) $1-\sqrt{3}$ (c) $1+\sqrt{3}$ (d) $-\sqrt{3}$

4. If $1-i$ is a root of the equation $x^2+ax+b=0$, where $a, b \in R$, then find the values of a and b.

(a) 2, 2 (b) −2, 2 (c) −2, −2 (d) 1, 2

5. Two cards are drawn at random one by one without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

(a) $\frac{21}{104}$ (b) $\frac{25}{102}$ (c) $\frac{23}{102}$ (d) $\frac{25}{104}$

6. The number of solutions of $\cos 2\theta = \sin\theta$ in $(0, 2\pi)$ is

(a) 1 (b) 2 (c) 3 (d) 4

7. If $\log_{10} 2 = 0.30103$, then $\log_{10} 50 =$

(a) 2.30103 (b) 2.69897 (c) 1.69897 (d) 0.69897

8. If (4, 0) is a point on the circle $x^2+ax+y^2=0$, then the centre of the circle is at

(a) (−2, 0) (b) (0, 2) (c) (2, 0) (d) (1, 0)

9. The integrating factor of the differential equation $(x+3y^2)\frac{dy}{dx}=y$ is

(a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

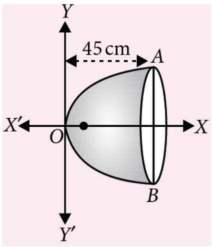
10. How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

(a) 90 (b) 50 (c) 40 (d) 100

11. If the set A contains 5 elements, then the number of elements in the power set $P(A)$ is

(a) 32 (b) 25 (c) 16 (d) 8

12. The focus of a parabolic mirror as shown in figure is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, find the distance AB.



(a) 30 cm
(b) 45 cm
(c) 60 cm
(d) 90 cm

13. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}-4\hat{k}, \vec{c}=\hat{i}+\lambda\hat{j}+3\hat{k}$ are coplanar, then the value of λ is

(a) 5/2 (b) 3/5 (c) 7/3 (d) None of these

14. If A, B, C, D be angles of a cyclic quadrilateral taken in order, then $\cos A + \cos B + \cos C + \cos D =$

(a) 0 (b) 1 (c) −1 (d) 1/2

15. Find the value of $\tan(\alpha+\beta)$, given that $\cot\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = \frac{-5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$.

(a) 1/11 (b) 2/11 (c) 5/11 (d) 3/11

16. The sum to 20 terms of the series $1\times 3^2 + 2\times 5^2 + 3\times 7^2 + \ldots\ldots$, is

(a) 18800 (b) 188010 (c) 188020 (d) 188090

17. If α and β are the roots of the equation $x^2+2x+4=0$, then $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ is equal to

(a) −1/2 (b) 1/2 (c) 32 (d) 1/4

18. In a ΔABC, a, c and A are given and b_1, b_2 are two values of the third side b such that $b_2 = 2b_1$. Then $\sin A =$

(a) $\sqrt{\frac{9c^2-a^2}{8a^2}}$ (b) $\sqrt{\frac{9a^2-c^2}{8c^2}}$

(c) $\sqrt{\frac{8a^2-9c^2}{8a^2}}$ (d) none of these

19. The differential equation satisfied by $y = \frac{A}{x} + B$ is (A, B are parameters)

(a) $x^2y_1 = y$ (b) $xy_1 + 2y_2 = 0$

(c) $xy_2 + 2y_1 = 0$ (d) none of these

20. Find a if the 17th and 18th terms of the expansion $(2 + a)^{50}$ are equal.

(a) 1 (b) 2 (c) 3 (d) 4

21. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

22. If $x^m y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x+y}{xy}$ (b) xy (c) $\frac{x}{y}$ (d) $\frac{y}{x}$

23. The function $f(x) = xe^{1-x}$

(a) strictly increases in the interval $(1/2, 2)$

(b) increases in the interval $(0, \infty)$

(c) decreases in the interval $(0, 2)$

(d) strictly decreases in the interval $(1, \infty)$

24. There are 7 greeting cards, each of a different colour and 7 envelopes of same 7 colours as that of the cards. The number of ways in which the cards can be put in envelopes, so that exactly 4 of the cards go into envelopes of respective colour, is

(a) ${}^7C_3$ (b) $2 \cdot {}^7C_3$

(c) $3!\ {}^4C_4$ (d) $3!\ {}^7C_3\ {}^4C_3$

25. If $\Delta = \begin{vmatrix} a+1 & a+2 & a+4 \\ a+3 & a+5 & a+8 \\ a+7 & a+10 & a+14 \end{vmatrix}$, then value of Δ is

(a) 2 (b) -2 (c) 4 (d) -4

26. The equation of circle of radius $\sqrt{17}$ units, with centre on the positive side of x-axis and through the point $(0, 1)$ is

(a) $x^2 + y^2 - 8x - 1 = 0$

(b) $x^2 + y^2 + 8x - 1 = 0$

(c) $x^2 + y^2 - 9y + 1 = 0$

(d) $2x^2 + 2y^2 - 3x + 2y = 4$

27. Let $g(x) = x^2 - 4x - 5$, then

(a) g is one-one on R (b) g is not one-one on R

(c) g is bijective on R (d) None of these

28. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

(a) $8:1$ (b) $1:8$ (c) $9:1$ (d) $1:9$

29. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) 1/2 (b) 1/3 (c) -1 (d) 1

30. The area between the curve $y = 4 + 3x - x^2$ and x-axis is

(a) $\frac{125}{6}$ sq. units (b) $\frac{125}{3}$ sq. units

(c) $\frac{125}{2}$ sq. units (d) none of these

31. If $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 676$ and $|\vec{b}| = 2$, then $|\vec{a}|$ is equal to

(a) 13 (b) 26

(c) 39 (d) None of these

32. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by

(a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

(c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

33. If A and B are two independent events, then the probability of occurrence of atleast one of A and B is given by

(a) $1 - P(A)\,P(B)$ (b) $1 - P(A)\,P(B')$

(c) $1 - P(A')\,P(B')$ (d) $1 - P(A')\,P(B)$

34. If $\frac{z_2}{z_1}$ is pure imaginary, then $\left|\frac{6z_1 - 8z_2}{4z_2 + 3z_1}\right| =$

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) 4

35. If in a triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, then $a : b : c =$

(a) $1 : 1 : \sqrt{2}$ (b) $1 : \sqrt{3} : 1$

(c) $1 : \sqrt{2} : 1$ (d) $\sqrt{2} : \sqrt{3} : 1$

36. The value of $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$ is

(a) $\pi/60$ (b) $\pi/20$ (c) $\pi/40$ (d) $\pi/80$

37. $\lim\limits_{x\to 0}\dfrac{\pi^x-1}{\sqrt{1+x}-1}$

(a) does not exist (b) equals $\log_e(\pi^2)$
(c) equals 1 (d) lies between 10 and 11

38. Find the 4th term from the end in the expansion of $\left(\dfrac{x^3}{2}-\dfrac{2}{x^2}\right)^9$.

(a) $\dfrac{674}{x}$ (b) $\dfrac{670}{x^3}$ (c) $\dfrac{672}{x^3}$ (d) $\dfrac{-670}{x^3}$

39. Find the point of intersection of the lines $\dfrac{x+1}{-3}=\dfrac{y-3}{2}=\dfrac{z+2}{1}$ and $\dfrac{x}{1}=\dfrac{y-7}{-3}=\dfrac{z+7}{2}$.

(a) (−2, 1, 3) (b) (2, 1, −3)
(c) (2, 3, 1) (d) (2, −1, 3)

40. The mean and variance of n observations x_1, x_2, x_3, ..., x_n are 5 and 0 respectively. If $\sum\limits_{i=1}^{n}x_i^2=400$, then the value of n is equal to

(a) 80 (b) 25 (c) 20 (d) 16

41. The solution of the differential equation $\dfrac{dy}{dx}=\dfrac{e^x+x^2}{e^y}$ is

(a) $e^x=\dfrac{y^3}{3}+e^y+c$ (b) $e^y=\dfrac{x^2}{3}+e^x+c$
(c) $e^y=\dfrac{x^3}{3}+e^x+c$ (d) None of these

42. A straight line through the point (3, −2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If it intersects the x-axis, then its equation will be

(a) $y+x\sqrt{3}+2+3\sqrt{3}=0$
(b) $y-x\sqrt{3}+2+3\sqrt{3}=0$
(c) $y-x\sqrt{3}-2-2\sqrt{3}=0$
(d) $y-x\sqrt{3}+2-3\sqrt{3}=0$

43. If x and 'a' are real, then the value of 'a' for which $x^2-\dfrac{3ax}{2}+1-a^2$ is positive is

(a) $-\dfrac{4}{25}p$ (b) $\dfrac{4}{25}$ (c) $|a|>\dfrac{4}{5}$ (d) $|a|<\dfrac{4}{5}$

44. If $P(n)$ is a statement such that $P(3)$ is true. Also $P(k)$ is true ⇒ $P(k+1)$ is true for all $k \geq 3$, then $P(n)$ is true

(a) for all $n \in N$ (b) for all $n \geq 3$, $n \in N$
(c) for all $n > 4$, $n \in N$ (d) none of these

45. If A and B are non-empty sets, then $(A \cap B) \cup (A - B)$ is equal to

(a) B (b) A (c) A' (d) B'

46. The second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t = \pi/4$ is

(a) 2 (b) $\dfrac{1}{12a}$ (c) $\dfrac{4\sqrt{2}}{3a}$ (d) $\dfrac{3a}{4\sqrt{2}}$

47. A function $f(x)$ is defined as follows for real x

$$f(x)=\begin{cases}1-x^2, & \text{for } x<1\\ 0, & \text{for } x=1\\ 1+x^2, & \text{for } x>1\end{cases}\text{, then}$$

(a) $f(x)$ is not continuous at $x = 1$
(b) $f(x)$ is continuous at $x = 1$
(c) $f(x)$ is continuous at all real numbers
(d) none of these

48. $\int \sin^{-1} x\,dx$ is equal to

(a) $\cos^{-1} x + C$ (b) $x\sin^{-1}x+\sqrt{1-x^2}+C$
(c) $\dfrac{1}{\sqrt{1-x^2}}+C$ (d) $x\sin^{-1}x-\sqrt{1-x^2}+C$

49. If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of conic $9x^2 - 4y^2 = 36$, then which of the following is true?

(a) $e_1^2 + e_2^2 = 2$ (b) $3 < e_1^2 + e_2^2 < 4$
(c) $e_1^2 + e_2^2 > 4$ (d) None of these

50. The determinant $\begin{vmatrix} y+k & y & y\\ y & y+k & y\\ y & y & y+k\end{vmatrix}$ is equal to

(a) $k(3y + k^2)$ (b) $3y + k^3$
(c) $3y + k^2$ (d) $k^2(3y + k)$

Category-II (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : ½)

51. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume.

(a) 2 cm, 430 cm³ (b) 5 cm, 425 cm³
(c) 3 cm, 432 cm³ (d) 3 cm, 423 cm³

52. If $\tan^{-1}\left(\dfrac{x+1}{x-1}\right)+\tan^{-1}\left(\dfrac{x-1}{x}\right)=\tan^{-1}(-7)$, then the value of x is

(a) 0 (b) −2 (c) 1 (d) 2

53. Find the standard deviation for the following data :

x_i	3	8	13	18	23
f_i	7	10	15	10	6

(a) 6.12 (b) 5.12 (c) 3.12 (d) 7.12

54. Evaluate : $\int_0^{\pi/2} \frac{1}{2\cos x + 4\sin x}dx$

(a) $\sqrt{5}\log\left(\frac{3+\sqrt{5}}{2}\right)$ (b) $\frac{1}{\sqrt{5}}\log\left(\frac{3-\sqrt{5}}{5}\right)$

(c) $\frac{1}{\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{2}\right)$ (d) None of these

55. If $y^{y^{y^{\cdots\cdots\infty}}} = \log_e(x + \log_e(x + \ldots..))$, then $\frac{dy}{dx}$ at $(x = e^2 - 2, y = \sqrt{2})$ is

(a) $\frac{\log\left(\frac{e}{2}\right)}{2\sqrt{2}(e^2-1)}$ (b) $\frac{\log 2}{2\sqrt{2}(e^2-1)}$

(c) $\frac{\sqrt{2}\log\frac{e}{2}}{(e^2-1)}$ (d) None of these

56. Coloured balls are distributed in three bags as shown in the following table :

Bag	Colour of balls		
	Black	White	Red
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are drawn from the selected bag. They happen to be black and red. What is the probability that they have come from bag I?

(a) $\frac{231}{551}$ (b) $\frac{232}{551}$ (c) $\frac{233}{550}$ (d) $\frac{231}{550}$

57. If ω is an imaginary cube root of unity and

$$\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0,$$ then one of the values of x is

(a) 1 (b) 0 (c) – 1 (d) 2

58. The vector equation of the plane passing through the point (–1, 2, –5) and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ is

(a) $\vec{r}\cdot(-2\hat{i} + 7\hat{j} + 5\hat{k}) = 13$

(b) $\vec{r}\cdot(-2\hat{i} + 7\hat{j} + 5\hat{k}) = -13$

(c) $\vec{r}\cdot(-2\hat{i} + 7\hat{j} + 5\hat{k}) = -9$

(d) $\vec{r}\cdot(-2\hat{i} + 7\hat{j} + 5\hat{k}) = 9$

59. If the tangent drawn at a point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as normal drawn at $(\sqrt{5}\cos\alpha, 2\sin\alpha)$ on the ellipse $\frac{x^2}{5} + \frac{y^2}{4} = 1$, then which of following is not true?

(a) $t = \pm 1/\sqrt{5}$ (b) $\alpha = -\tan^{-1}2$

(c) $\alpha = \tan^{-1}2$ (d) None of these

60. If a, b, c be the p^{th}, q^{th} and r^{th} terms respectively of an A.P.and G.P. both, then the product of the roots of equation $(a^b b^c c^a)x^2 - (abc)x + (a^c b^a c^b) = 0$ equals

(a) – 1 (b) 2 (c) abc (d) 1

61. Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

(a) $\frac{7}{9}\left[\frac{10(10^n-1)}{9} - n\right]$ (b) $\frac{7}{9}\left[\frac{10(10^n+1)}{9} - n\right]$

(c) $\frac{10^n-1}{9}$ (d) None of these

62. Let $L = \lim_{x\to 0}\frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

(a) $a = 2$ (b) $a = 1$

(c) $a = 1/3$ (d) None of these

63. Find the smaller of the two areas enclosed by the curves $x^2 + y^2 = 4$ and $y^2 = 3(2x - 1)$.

(a) $(4\pi - \sqrt{3})$ sq. units (b) $\frac{1}{2}(2\pi + \sqrt{3})$ sq. units

(c) $\frac{1}{3}(4\pi - \sqrt{3})$ sq. units (d) None of these

64. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then

(a) both $\rho_1 \cap \rho_2$ and $\rho_1 \cup \rho_2$ are equivalence relations.

(b) $\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so.

(c) $\rho_1 \cup \rho_2$ is equivalence relation but $\rho_1 \cap \rho_2$ is not so.

(d) neither $\rho_1 \cap \rho_2$ nor $\rho_1 \cup \rho_2$ is equivalence relation.

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(a)	**4.**	(d)	**5.**	(b)
6.	(b)	**7.**	(b,c)	**8.**	(a,b,d)	**9.**	(a,b)	**10.**	(a,b,c)
11.	(a,b,c)	**12.**	(a,c)	**13.**	(a,b,c)	**14.**	(a)	**15.**	(b)
16.	(d)	**17.**	(5)	**18.**	(1)	**19.**	(3)	**20.**	(2)

65. Evaluate : $\displaystyle\int \frac{x^2}{(x-1)^3(x+1)}dx$

(a) $\frac{1}{8}\log\left|\frac{x-1}{x+1}\right|+\frac{3}{4(x-1)}+\frac{1}{4(x-1)^2}+C$

(b) $\frac{1}{8}\log\left|\frac{x+1}{x-1}\right|-\frac{3}{4(x-1)}-\frac{1}{4(x+1)^2}+C$

(c) $\frac{1}{8}\log\left|\frac{x-1}{x+1}\right|-\frac{3}{4(x-1)}-\frac{1}{4(x-1)^2}+C$

(d) None of these

Category-III (Q.66 to 75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. If maximum and minimum values of the determinant $\begin{vmatrix} 1+\cos^2 x & \sin^2 x & \cos 2x \\ \cos^2 x & 1+\sin^2 x & \cos 2x \\ \cos^2 x & \sin^2 x & 1+\cos 2x \end{vmatrix}$ are α and β, then

(a) $\alpha^2 + \beta^{101} = 10$ (b) $\alpha^3 - \beta^{99} = 26$

(c) $2\alpha^2 - 18\beta^{11} = 0$ (d) $\alpha^3 + 2\beta^2 = 0$

67. The internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD meets the side AC at E and the side AB at F. If a,b,c, are the sides of triangle ABC, then

(a) AE is H.M. of b and c

(b) $AD=\frac{2bc}{b+c}\cos\frac{A}{2}$ (c) $EF=\frac{4bc}{b+c}\sin\frac{A}{2}$

(d) triangle AEF is isosceles

68. A and B are independent events. The probability that both A and B occur is 1/20 and the probability that neither of them occurs is 3/5. The probability of occurrence of A is

(a) 1/2 (b) 1/10 (c) 1/4 (d) 1/5

69. If $\frac{\cos x-\cos\alpha}{\cos x-\cos\beta}=\frac{\sin^2\alpha\cos\beta}{\sin^2\beta\cos\alpha}$, then

(a) $\cos x=\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}$

(b) $\cos x=\frac{\cos\alpha+\cos\beta}{1-\cos\alpha\cos\beta}$

(c) $\tan\frac{x}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$

(d) $\tan\frac{x}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$

70. Equation of a tangent to the hyperbola $5x^2 - y^2 = 5$ and which passes through an external point (2, 8) is

(a) $3x - y + 2 = 0$ (b) $3x + y - 14 = 0$

(c) $23x - 3y - 22 = 0$ (d) $3x - 23y + 178 = 0$

71. Given a real valued function f such that

$$f(x)=\begin{cases} \dfrac{\tan^2 x}{(x^2-[x]^2)}, & \text{for } x>0 \\ 1, & \text{for } x=0 \\ \sqrt{\{x\}\cot\{x\}}, & \text{for } x<0 \end{cases}$$

where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then

(a) $\lim_{x\to 0^+} f(x)=1$ (b) $\lim_{x\to 0^-} f(x)=\sqrt{\cot 1}$

(c) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2=1$

(d) f is continuous at $x = 0$

72. Let $f(x)$ be twice differentiable function such that $f''(x) > 0$ in [0, 1]. Then

(a) $f(0) + f(1) = 2f(c), 0 < c < 1$

(b) $f(0) + f(1) = 2f\left(\frac{1}{2}\right)$

(c) $f(0) + f(1) > 2f\left(\frac{1}{2}\right)$

(d) $f(0) + f(1) < 2f\left(\frac{1}{2}\right)$

73. Which of the following is/are the point/points that is/are at a distance of 12 units from the point whose position vector is $\left(8\hat{i}+10\hat{j}-8\hat{k}\right)$ on the line which is parallel to $\left(2\hat{i}+\hat{j}+2\hat{k}\right)$?

(a) $16\hat{i}+14\hat{j}$ (b) $6\hat{j}-16\hat{k}$

(c) $16\hat{i}+18\hat{j}-4\hat{k}$ (d) None of these

74. Three vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\times\vec{b} = 4(\vec{a}\times\vec{c})$ and $|\vec{a}|=|\vec{b}| = 1$ and $|\vec{c}| = \frac{1}{4}$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then $\vec{b}$ is

(a) $\vec{a}+4\vec{c}$ (b) $\vec{a}-4\vec{c}$ (c) $4\vec{c}-\vec{a}$ (d) $2\vec{c}-\vec{a}$

75. Tangent is drawn at any point $P(x, y)$ on a curve, which passes through (1, 1). The tangent cuts X-axis and Y-axis at A and B respectively. If $AP : BP = 3 : 1$, then

(a) the differential equation of the curve is

$$3x\frac{dy}{dx}+y=0$$

(b) the differential equation of the curve is

$$3x\frac{dy}{dx} - y = 0$$

(c) the curve passes through $\left(\frac{1}{8}, 2\right)$

(d) the normal at (1, 1) is $x + 3y = 4$

SOLUTIONS

1. (a) : Reflexive : For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ is an irrational number $\Rightarrow xRx$ for all x. So, R is reflexive.

Symmetric : R is not symmetric, because $\sqrt{2} R 1$ but $1 \not R \sqrt{2}$.

Transitive : R is not transitive also, because $\sqrt{2} R 1$ and $1 R 2\sqrt{2}$ but $\sqrt{2} \not R 2\sqrt{2}$.

2. (c) : Since the sum is 28, we take the A.P. as $7 - 3d, 7 - d, 7 + d, 7 + 3d$.

It is given that $\frac{(7-3d)(7+d)}{(7-d)(7+3d)} = \frac{8}{15}$

$\Rightarrow$ $15[49 - 14d - 3d^2] = 8[49 + 14d - 3d^2]$

$\Rightarrow$ $3d^2 + 46d - 49 = 0 \Rightarrow d = 1$

$\therefore$ The four parts are 4, 6, 8, 10.

3. (a) : Since, $\vec{a} + \vec{b} = \vec{c} \Rightarrow (\vec{a} + \vec{b})^2 = \vec{c}^{\,2}$

$\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow$ $2(1 + \cos\theta) = 1 \Rightarrow \cos\theta = -\frac{1}{2}$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$= 1 + 1 - 2\cdot\left(\frac{-1}{2}\right) = 3 \Rightarrow |\vec{a} - \vec{b}| = \sqrt{3}$

4. (b) : Since complex roots always occur in conjugate pair.

$\therefore$ Other root is $1 + i$.

Sum of roots $= \frac{-a}{1} = (1 - i) + (1 + i) \Rightarrow a = -2$

Product of roots $= \frac{b}{1} = (1 - i)(1 + i) \Rightarrow b = 2$

5. (b) : Let E and F be the events that first and second card drawn are black, respectively.

Then, $P(E) = P(\text{black card in first draw}) = \frac{26}{52} = \frac{1}{2}$

When second card is drawn without replacement, the probability that the second card is black is the conditional probability of event F occurring when event E has already occurred.

So, $P(F \mid E) = \frac{25}{51}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\cdot P(F \mid E) = \frac{1}{2}\cdot\frac{25}{51} = \frac{25}{102}$

6. (c) : $\cos 2\theta = \sin\theta \Rightarrow 1 - 2\sin^2\theta - \sin\theta = 0$

$\Rightarrow$ $2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

$\Rightarrow$ $2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

$\Rightarrow$ $\sin\theta = -1, 1/2$

In $(0, 2\pi)$; $\sin\theta = -1$ for $\theta = 3\pi/2$

and $\sin\theta = \frac{1}{2}$ for $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ Total 3 solutions exist in $(0, 2\pi)$.

7. (c) : Consider $\log_{10} 50 = \log_{10}\frac{100}{2}$

$= \log_{10} 10^2 - \log_{10} 2 = 2\log_{10} 10 - \log_{10} 2 = 2 - (0.30103)$

$= 1.69897$

8. (c) : Since, (4, 0) is a point on the circle $x^2 + ax + y^2 = 0$.

$\therefore$ $16 + 4a + 0 = 0 \Rightarrow 4a = -16 \Rightarrow a = -4$

$\therefore$ Equation of circle is $x^2 - 4x + y^2 = 0$

or $x^2 - 4x + 4 - 4 + y^2 = 0 \Rightarrow (x - 2)^2 + (y - 0)^2 = 4$

$\therefore$ Centre (2, 0) and radius = 2

9. (c) : We have, $(x + 3y^2)\frac{dy}{dx} = y$

$\Rightarrow$ $\frac{x + 3y^2}{y} = \frac{dx}{dy} \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

This is a linear differential equation.

$\therefore$ I.F. $= e^{-\int\frac{dy}{y}} = e^{-\log y} = e^{\log y^{-1}} = \frac{1}{y}$

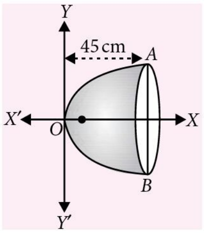
10. (d) : Every number between 100 and 1000 is a 3-digit number. We first have to count the permutations of 6 digits taken 3 at a time *i.e.*, ${}^6P_3$. But, these permutations will include those also where 0 is at the 100's place. For example, 092, 042, ..., etc are such numbers which are actually 2-digit numbers. To get the number of such numbers, we fix 0 at the 100's place and rearrange the remaining 5 digits taking 2 at a time. This will give ${}^5P_2$ numbers.

So, the required numbers $= {}^6P_3 - {}^5P_2 = \frac{6!}{3!} - \frac{5!}{3!} = 100$

11. (a) : $n(A) = 5 \Rightarrow n(P(A)) = 2^5 = 32$.

12. (c) : Since the distance from the focus to the vertex is 5 cm, we have, $a = 5$. Also, the axis of the mirror lies along the positive x-axis, the equation of the parabolic section is

$$y^2 = 4(5)x = 20x$$



When $x = 45$, $y^2 = 900$

Therefore, $y = \pm 30$. Hence, $AB = 2y = 2 \times 30 = 60$ cm

13. (d) : Since, $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}-4\hat{k},\ \vec{c}=\hat{i}+\lambda\hat{j}+3\hat{k}$ are coplanar.

$$\therefore\quad \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -4 \\ 1 & \lambda & 3 \end{vmatrix}=0$$

$\Rightarrow\quad 4\lambda - 1(6 + 4) + 2\lambda = 0 \Rightarrow \lambda = 5/3$

14. (a) : In a cyclic quadrilateral, the sum of opposite angles is 180°.

$\therefore\quad A + C = 180°$ and $B + D = 180°$

$\Rightarrow\quad C = 180° - A$ and $D = 180° - B$ ...(i)

Now, $\cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B + \cos(180° - A) + \cos(180° - B)$ [From (i)]

$= \cos A + \cos B - \cos A - \cos B = 0$

15. (b) : Given, $\cot\alpha = \dfrac{1}{2} \Rightarrow \tan\alpha = 2$ and $\sec\beta = \dfrac{-5}{3}$

Then, $\tan\beta = \sqrt{\sec^2\beta - 1} \Rightarrow \tan\beta = \pm\dfrac{4}{3}$

But, $\tan\beta = \dfrac{-4}{3}$ $(\because\ \tan\beta$ is –ve in II quadrant)

$$\therefore\quad \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} = \frac{2+\left(-\frac{4}{3}\right)}{1-(2)\left(\frac{-4}{3}\right)} = \frac{2}{11}$$

16. (d) : The r^{th} term of the series $t_r = r(2r+1)^2$
$= 4r^3 + 4r^2 + r$

$$\sum_{r=1}^{20} t_r = 4\sum_{r=1}^{20} r^3 + 4\sum_{r=1}^{20} r^2 + \sum_{r=1}^{20} r$$

$$= 4\cdot\left(\frac{20\times 21}{2}\right)^2 + \frac{4\cdot 20\cdot 21\cdot 41}{6} + \frac{20\cdot 21}{2}$$

$= 4(210)^2 + 40 \cdot 7 \cdot 41 + 210 = 188090$

17. (d) : Given equation is $x^2 + 2x + 4 = 0$

Since α, β are roots of this equation.

$\therefore\quad \alpha + \beta = -2$ and $\alpha\beta = 4$

$$\text{Now, } \frac{1}{\alpha^3}+\frac{1}{\beta^3} = \frac{\alpha^3+\beta^3}{(\alpha\beta)^3} = \frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)^3}$$

$$= \frac{(-2)((\alpha+\beta)^2-3\alpha\beta)}{4\times4\times4} = \frac{-2(4-12)}{4\times4\times4} = \frac{(-2)\times(-8)}{4\times4\times4} = \frac{1}{4}$$

18. (b) : $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\quad b^2 - 2bc\cos A + (c^2 - a^2) = 0$

As, b_1 and b_2 are two values of b.

$\therefore\quad b_1 + b_2 = 2c\cos A$ and $b_1 b_2 = c^2 - a^2$

$\Rightarrow 3b_1 = 2c\cos A$ and $2b_1^2 = c^2 - a^2$ $[\because\ b_2 = 2b_1$ (given)]

$$\therefore\ 2\left(\frac{2c\cos A}{3}\right)^2 = c^2 - a^2 \Rightarrow 8c^2(1-\sin^2 A) = 9c^2 - 9a^2$$

$$\Rightarrow\ \sin^2 A = \frac{9a^2-c^2}{8c^2} \Rightarrow \sin A = \sqrt{\frac{(9a^2-c^2)}{8c^2}}$$

19. (c) : Given relation is $y = \dfrac{A}{x} + B$

Differentiating w.r.t. x, we get

$y_1 = -\dfrac{A}{x^2} \Rightarrow x^2 y_1 = -A$

Again, differentiating w.r.t. x, we get

$x^2y_2 + y_1 2x = 0 \Rightarrow xy_2 + 2y_1 = 0$

20. (a) : $T_{17} = T_{16+1} = {}^{50}C_{16}(2)^{50-16}a^{16} = {}^{50}C_{16}2^{34}a^{16}$

Similarly, $T_{18} = T_{17+1} = {}^{50}C_{17}\,2^{33}a^{17}$

Given that $T_{17} = T_{18}$

So, ${}^{50}C_{16}(2)^{34}a^{16} = {}^{50}C_{17}(2)^{33}a^{17}$

$$\text{Therefore } \frac{{}^{50}C_{16}\cdot 2^{34}}{{}^{50}C_{17}\cdot 2^{33}} = \frac{a^{17}}{a^{16}}$$

$$\Rightarrow\quad a = \frac{{}^{50}C_{16}\times 2}{{}^{50}C_{17}} = \frac{50!}{16!\times 34!}\times\frac{17!\times 33!}{50!}\times 2 = 1$$

21. (a) : Let $I = \displaystyle\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x} = \int_{-\pi/4}^{\pi/4}\frac{dx}{2\cos^2 x}$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx = \int_0^{\pi/4}\sec^2 x\,dx$$

$$\left[\text{Using property for even function } f(x),\ \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx\right]$$

$$= \left[\tan x\right]_0^{\pi/4} = \left[\tan\frac{\pi}{4} - \tan 0\right] = 1$$

22. (d) : Given, $x^m y^n = (x+y)^{m+n}$

Taking log on both sides, we get

$m\log x + n\log y = (m+n)\log(x+y)$

Differentiating w.r.t. x, we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\times\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\quad \frac{m}{x} - \frac{m+n}{x+y} = \left(\frac{m+n}{x+y} - \frac{n}{y}\right)\frac{dy}{dx}$$

$$\Rightarrow\quad \frac{my-nx}{x(x+y)} = \left(\frac{my-nx}{y(x+y)}\right)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

23. (d) : We have $f(x) = x\,e^{1-x}$

$f'(x) = -xe^{1-x} + e^{1-x} = (1-x)e^{1-x}$

If $f'(x) < 0 \Rightarrow (1-x)e^{1-x} < 0$

$\Rightarrow 1-x<0 \, (\because e^{1-x} > 0)$

$\Rightarrow x>1 \Rightarrow x \in (1, \infty)$ so function is strictly decreasing.

24. (b) : Number of ways in which exactly 4 out of 7 cards go into envelopes of respective colour is equal to selecting 3 cards out of 7 which is equal to ${}^7C_3$ and then dearranging them *i.e.,* none of the 3 cards go into its respective envelope.

$\therefore$ Required ways $= {}^7C_3 \times 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right]$

$= {}^7C_3 \times 3!\left[\frac{1}{2}-\frac{1}{6}\right] = {}^7C_3 \times 6 \times \frac{1}{3} = 2 \cdot {}^7C_3$

25. (b) : Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} a+1 & 1 & 3 \\ a+3 & 2 & 5 \\ a+7 & 3 & 7 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} a+1 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = 0-(8-12)+3(4-6) = -2$$

26. (a) : Let the centre of the circle is $(h, 0)$, where $h > 0$.

Now, circle passes through $(0, 1)$

$\therefore \quad r^2 = (h-0)^2 + (0-1)^2$

$\Rightarrow \quad (\sqrt{17})^2 = h^2 + 1$

$\Rightarrow \quad h^2 = 16 \Rightarrow h = \pm 4$

$\Rightarrow \quad h = 4 \quad (\because h$ can't be negative)

$\therefore \quad$ Equation of circle is $(x-4)^2 + (y-0)^2 = 17$

$\Rightarrow \quad x^2 - 8x + y^2 + 16 - 17 = 0 \Rightarrow x^2 + y^2 - 8x - 1 = 0$

27. (b) : Let $g(x_1) = g(x_2)$

$\Rightarrow \quad x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5 \Rightarrow \quad x_1^2 - x_2^2 = 4(x_1 - x_2)$

$\Rightarrow \quad (x_1 - x_2)(x_1 + x_2 - 4) = 0$

Either $x_1 = x_2$ or $x_1 + x_2 = 4$

Either $x_1 = x_2$ or $x_1 = 4 - x_2$

$\therefore$ There are two values of x for which $g(x_1) = g(x_2)$.

$\therefore \; g(x)$ is not one-one $\forall \; x \in R$

28. (a) : Let e_1 and e_2 be the eccentricity of given hyperbola and ellipse respectively.

$\therefore \quad e_1 = \sqrt{1+\frac{b^2}{a^2}}$ and $e_2 = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}}$

Given, $e_1 = \frac{1}{e_2} \Rightarrow 1+\frac{b^2}{a^2} = \frac{9}{8} \Rightarrow \frac{a^2}{b^2} = \frac{8}{1}$

29. (d) : We have, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right] = \sin\left(\frac{\pi}{2}\right) = 1$

30. (a) : We have, $y = 4 + 3x - x^2$, a parabola with vertex at $\left(\frac{3}{2}, \frac{25}{4}\right)$

Putting $y = 0$, we get $x^2 - 3x - 4 = 0$

$\Rightarrow \quad (x-4)(x+1) = 0 \Rightarrow x = -1$ or $x = 4$

$\therefore \quad$ Required area $= \int_{-1}^{4} (4+3x-x^2)dx$

$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^{4} = \frac{125}{6}$ sq. units

31. (a) : Since, $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 676$

$\Rightarrow \quad (|\vec{a}|\cdot|\vec{b}|\sin\theta\hat{n})^2 + (|\vec{a}|\cdot|\vec{b}|\cos\theta)^2 = 676$

$\Rightarrow \quad |\vec{a}|^2 \cdot |\vec{b}|^2 (\sin^2\theta + \cos^2\theta) = 676$

$\Rightarrow \quad |\vec{a}|^2 (2)^2 = 676 \Rightarrow |\vec{a}|^2 = 169 \Rightarrow |\vec{a}| = 13$

32. (b) : We have, $|A| = 6 + 1 = 7$

Also, $\text{adj } A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore \quad 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

33. (c) : P(atleast one of A and B)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B) \quad [\because A, B$ are independent$]$

$= P(A) + P(B)\,[1 - P(A)] = [1 - P(A')] + P(B)\,P(A')$

$= 1 - P(A') + P(B)\,P(A') = 1 - P(A')\,[1 - P(B)]$

$= 1 - P(A')\,P(B')$

34. (c) : $\frac{z_2}{z_1} = ik$, k is real, $z_2 = ikz_1$

$\Rightarrow \quad \frac{6z_1 - 8z_2}{4z_2 + 3z_1} = \frac{6z_1 - 8ikz_1}{4ikz_1 + 3z_1} = \frac{6-8ki}{4ik+3}$

$\therefore \quad \left|\frac{6z_1 - 8z_2}{4z_2 + 3z_1}\right| = \frac{|6-8ik|}{|4ik+3|} = \frac{\sqrt{36+64k^2}}{\sqrt{16k^2+9}} = \sqrt{4} = 2$

35. (a) : From the given relation

$\sin C = \frac{1-\cos A\cos B}{\sin A\sin B} \le 1 \qquad$ ...(i)

$\Rightarrow 1 \le \cos A \cos B + \sin A \sin B$

$\Rightarrow 1 \le \cos(A-B)$ but $\cos(A-B) \le 1$

$\Rightarrow \cos(A-B) = 1 \Rightarrow A - B = 0 \Rightarrow A = B$ ...(ii)

From (i) and (ii), we get, $\sin C = \frac{1-\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$

$\therefore\ \angle C = 90°$

$\Rightarrow\ A + B = 90° \Rightarrow A = B = 45°$ [Using (ii)]

$\therefore\ a : b : c = \sin A : \sin B : \sin C = \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{2}} : 1 = 1 : 1 : \sqrt{2}$

36. (a) : We have,

$$\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \frac{1}{5}\int_0^\infty \left[\frac{1}{x^2+4} - \frac{1}{x^2+9}\right] dx$$

$$= \frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^\infty - \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty\right]$$

$$= \frac{1}{5}\left[\frac{1}{2}\tan^{-1}\infty - \frac{1}{3}\tan^{-1}\infty\right] = \frac{1}{5}\left[\frac{\pi}{4} - \frac{\pi}{6}\right] = \frac{\pi}{60}$$

37. (b) : $\lim\limits_{x\to 0} \frac{\pi^x - 1}{\sqrt{1+x} - 1}$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x\to 0} \frac{\pi^x \log_e \pi}{\frac{1}{2\sqrt{1+x}} - 0} \quad \text{[Using L'Hospital's rule]}$$

$$= 2\log_e \pi = \log_e(\pi^2)$$

38. (c) : 4th term from the end is 9 – 4 + 2, *i.e.*, 7th term from the beginning.

$$\therefore\quad T_7 = {}^9C_6\left(\frac{x^3}{2}\right)^3\left(\frac{-2}{x^2}\right)^6 = {}^9C_3\frac{x^9}{8}\cdot\frac{64}{x^{12}}$$

$$= \frac{9\times 8\times 7}{3\times 2\times 1}\times\frac{8}{x^3} = \frac{672}{x^3}$$

39. (b) : Let $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} = \lambda$

$\therefore$ The coordinates of a point on the line are $(-3\lambda - 1, 2\lambda + 3, \lambda - 2)$.

Similarly, let $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2} = \mu$

The coordinates of a point on this line are $(\mu, -3\mu + 7, 2\mu - 7)$.

Since, two lines intersect for same values of λ and μ,

$\therefore\ (-3\lambda - 1, 2\lambda + 3, \lambda - 2) = (\mu, -3\mu + 7, 2\mu - 7)$

$\Rightarrow\ -3\lambda - 1 = \mu,\ 2\lambda + 3 = -3\mu + 7,\ \lambda - 2 = 2\mu - 7$

$\therefore\ -3\lambda - \mu = 1$ ...(i)

$2\lambda + 3\mu = 4$ ...(ii)

$\lambda - 2\mu = -5$ ...(iii)

Solving equation (i) and (ii), we get $\lambda = -1$ and $\mu = 2$ and 3rd equation holds for these values.

$\therefore$ The lines intersect at the point (2, 1, –3).

40. (d) : Variance = $\sigma^2 \Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2 = \sigma^2$

$$\Rightarrow\ 0 = \frac{1}{n}(400) - (5)^2 \Rightarrow \frac{400}{n} = 25 \Rightarrow n = 16$$

41. (c) : From given differential equation, we have

$$\frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

Using variable separable form, we have

$e^y dy = (e^x + x^2)\, dx$

On integrating, we get $e^y = e^x + x^3/3 + c$

42. (b) : Let the slope of line through the point (3, – 2) be m.

Now, slope of line $\sqrt{3}x + y = 1$ is $-\sqrt{3}$.

$$\therefore\ \tan 60° = \left|\frac{-\sqrt{3} - m}{1 + (-\sqrt{3})m}\right| \Rightarrow \sqrt{3} = \left|\frac{\sqrt{3} + m}{1 - \sqrt{3}\,m}\right|$$

$$\Rightarrow\ \pm\sqrt{3} = \frac{\sqrt{3} + m}{1 - \sqrt{3}m} \Rightarrow\ m = \sqrt{3}, 0$$

$\therefore$ Equations of lines passing through (3, –2) and having slopes $\sqrt{3}$, 0 are $y + 2 = \sqrt{3}(x - 3)$ and $y + 2 = 0$ (rejected) $\Rightarrow\ y - \sqrt{3}x + 3\sqrt{3} + 2 = 0$

43. (d) : Given, $x^2 - \frac{3ax}{2} + 1 - a^2$ is positive.

$$\Rightarrow\ x^2 - \frac{3ax}{2} + 1 - a^2 > 0$$

$\therefore\ 9a^2 - 4(2)(2 - 2a^2) < 0$ ($\because$ If $f(x) > 0$ $\therefore$ $D < 0$)

$\Rightarrow\ 9a^2 - 16 + 16a^2 < 0 \Rightarrow 25a^2 - 16 < 0$

$$\Rightarrow\ 25a^2 < 16 \Rightarrow a^2 < \frac{16}{25} \Rightarrow |a| < \frac{4}{5}$$

44. (b) : Since, $P(3)$ is true and $P(k)$ is true

$\Rightarrow$ $P(k + 1)$ is true, which means, if $P(3)$ is true

$\Rightarrow$ $P(4)$ is true $\Rightarrow P(5)$ is true and so on. So statement is true for all $n \geq 3$, $n \in N$.

45. (b) : $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$ [$\because (A - B) = (A \cap B')$]

$= A \cap (B \cup B')$ (By distributive law)

$= A \cap U = A$ ($\because B \cup B' = U$)

46. (c) : Let $y = a\sin^3 t$ and $x = a\cos^3 t$

$$\therefore\ \frac{dy}{dt} = 3a\sin^2 t\cos t \text{ and } \frac{dx}{dt} = -3a\cos^2 t\sin t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{-\cos t} = -\tan t$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sec^2 t\cdot\frac{dt}{dx} = \frac{-\sec^2 t}{-3a\cos^2 t\sin t} = \frac{1}{3a\cos^4 t\sin t}$$

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{t=\pi/4} = \left(\frac{1}{3a}\frac{1}{\cos^4 t \sin t}\right)_{t=\pi/4} = \frac{4\sqrt{2}}{3a}.$$

47. (a) : To check continuity at $x = 1$

$$\text{L.H.L.} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0}\left(1-(1-h)^2\right) = 1 - 1 = 0$$

$$\text{R.H.L.} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0}\left(1+(1+h)^2\right) = 1 + 1 = 2$$

$\therefore$ L.H.L. $\neq$ R.H.L. $\neq f(1)$

$\therefore$ $f(x)$ is not continuous at $x = 1$

48. (b) : Let $I = \int 1\cdot\sin^{-1} x\,dx$

$$= (\sin^{-1}x)x - \int \frac{1}{\sqrt{1-x^2}}\cdot x\,dx + C'$$

Putting $1 - x^2 = t^2 \Rightarrow -2x\,dx = 2t\,dt$, we get

$$I = x\sin^{-1}x - \int\frac{(-t\,dt)}{t} + C' = x\sin^{-1}x + \sqrt{1-x^2} + C$$

49. (b) : From given equations, we have

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \text{ and } \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\therefore \quad e_1^2 = \frac{9-4}{9} \text{ and } e_2^2 = \frac{4+9}{4}$$

$$\text{Now, } e_1^2 + e_2^2 = \left(\frac{5}{9}\right) + \left(\frac{13}{4}\right) = \frac{137}{36} > 3 \text{ but } < 4$$

$\therefore$ $3 < e_1^2 + e_2^2 < 4$

50. (d) : Let $D = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

$= (y + k)((y + k)^2 - y^2) - y(y^2 + ky - y^2) + y(y^2 - y^2 - yk)$

$= k^3 + 3k^2y = k^2(k + 3y)$

51. (c) **52. (d)** **53. (a)**
54. (c) **55. (a)** **56. (a)**
57. (b) **58. (c)** **59. (d)**
60. (d) **61. (a)** **62. (a)**
63. (c) **64. (b)** **65. (c)**
66. (a, b, c) **67. (a, b, c, d)** **68. (c, d)**
69. (a, c, d) **70. (a, c)** **71. (a, b, c)**
72. (a, c) **73. (a, b)** **74. (a, c)**
75. (a, c)

◆ ◆

UNSCRAMBLE ME

Unscramble the words given in column I and match them with their explanations in column II.

Column I		Column II
1. COCAPIRREL	(a)	A statistical technique that relates a dependent variable to one or more independent variable.
2. TILEPERNEC	(b)	In probability theory and statistics it is used to measure the joint variability of two random variables.
3. VINTNAIRA	(c)	A straight line that is neither horizontal nor vertical.
4. OILBUEQ	(d)	The numbers resulting from the division of one number by another.
5. ERESONISGR	(e)	The multiplicative inverse of a value or a number.
6. VINALTER	(f)	A set that contains all the elements of another set.
7. RIVELTIAMULBA	(g)	A specific set of real numbers that lies between two conditional numbers of the set.
8. ANARICOCEV	(h)	A function, quantity which remains unchanged when specified transformation is applied.
9. UNTIQOET	(i)	Two or more variables.
10. SUTESREP	(j)	It is used in statistics indicating the value below which a given percentage of observations in a group of observations fall.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names will be published in next issue.

Are You Ready for *JEE?*

Top Weightage Topics for JEE **Series 1**

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

Sets

A well defined collection of distinct objects.

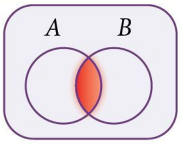
Representation of Sets

Set builder form or rule method:
$A = \{x : x \text{ is a prime number}\}$
Roster or tabular form :
$A = \{1, 2, 3, 4, 5\}$.

Types of Sets

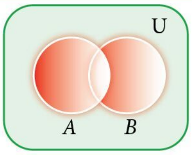
Empty or null set : $A = \phi$ or $\{\ \}$
Singleton set : $A = \{a\}$
Finite set : $A = \{1, 2, 3, 4, 5\}$
Infinite sets : $A = \{1, 2, 3, 4, \ldots\}$
Equal sets : $A = \{1, 2, 3, 4\}$
$B = \{3, 1, 4, 2\} \Rightarrow A = B$
Equivalent set : $A = \{1, 2, 3\}$ and $B = \{2, 5, 6\} \Rightarrow n(A) = n(B)$
Power set : $A = \{1, 2, 3\}$
$P(A) = \{\{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\}, \{3\}, \phi\}$
Disjoint sets : $A \cap B = \phi$

Intersection of two sets



$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Union of two sets



$A \cup B = \{x : x \in A \text{ or } x \in B\}$

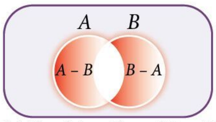
Demorgan's law

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Order/Cardinality

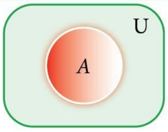
If B = {Red, Blue, White}, then total number of element of B, *i.e.*, $n(B) = 3$

Symmetric difference of A and B



$A \Delta B = (A - B) \cup (B - A)$
$x \in A \cup B$ but $x \in A \cap B$

Compliment of a Set



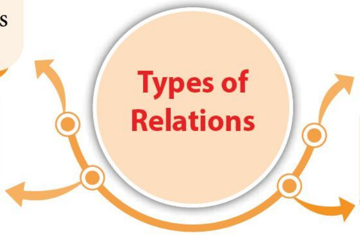
A' or $A^C = U - A$

Relations

If A and B are two non empty sets, then relation R from A to B is a subset of A × B.

Reflexive : A relation R on a set A is reflexive if $(a, a) \in R \,\forall\, a \in A$.

Symmetric : A relation R on a set A is symmetric if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R \,\forall\, a_1, a_2 \in A$.



Transitive : A relation R on a set A is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R \,\forall\, a_1, a_2, a_3 \in A$.

Equivalence : A relation R is an equivalence relation if R is reflexive, symmetric and transitive.

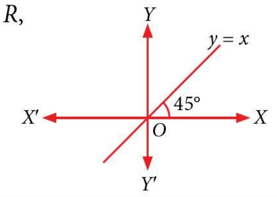
Functions

A relation 'f ' from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

Identity Function

$y = f(x) = x \ \forall\ x \in R,$

$D_f = R = R_f$



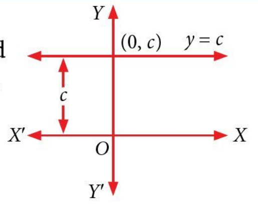
Constant Function

$y = f(x) = c,$

$\forall\ x \in R$, where fixed number c is called a constant.

$D_f = R$

$R_f = \{c\}$

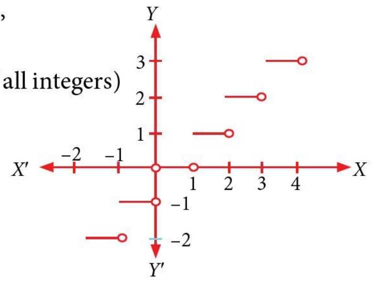

Y
(0, c)
y = c
c
X′
X
O
Y′

Greatest Integer Function

$y = f(x) = [x],$

$D_f = R$

$R_f = Z$ (Set of all integers)

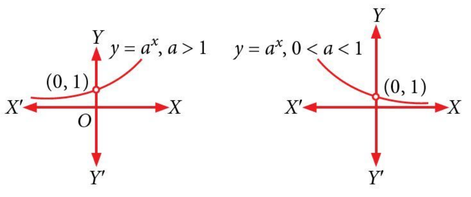

Y
3
2
1
X′
–2
–1
1
2
3
4
X
–1
–2
Y′

Exponential Function

$y = f(x) = a^x, a > 0, a \neq 1,$

$D_f = R,$

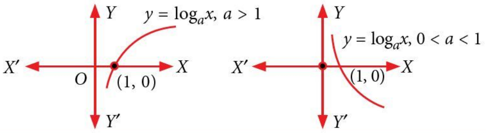
$R_f = (0, \infty)$


Y
y = a^x, a > 1
(0, 1)
X′
X
O
Y′
Y
y = a^x, 0 < a < 1
(0, 1)
X′
X
Y′

Logarithmic Function

$y = f(x) = \log_a x,\ a > 0,\ a \neq 1$ and $x > 0$

$D_f = (0, \infty),\ R_f = R.$


Y
y = log_a x, a > 1
X′
O
(1, 0)
X
Y′
Y
y = log_a x, 0 < a < 1
X′
(1, 0)
X
Y′

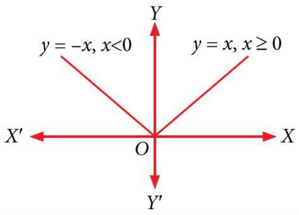
Modulus (Absolute Value) Function

$y = f(x) = |x|$, where

$$|x| = \begin{Bmatrix} x, & \text{if } x \geq 0 \\ -x & \text{if } \leq 0 \end{Bmatrix}$$

$D_f = R$

$R_f = [0, \infty)$


Y
y = –x, x<0
y = x, x ≥ 0
X′
X
O
Y′

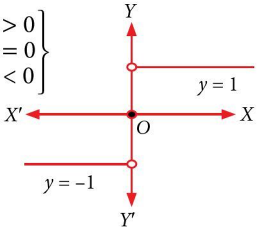
Signum Function

$y = f(x) = \text{sgn}\,(x),$

$$\text{where } f(x) = \begin{Bmatrix} 1, & \text{if } & x > 0 \\ 0, & \text{if } & x = 0 \\ -1, & \text{if } & x < 0 \end{Bmatrix}$$

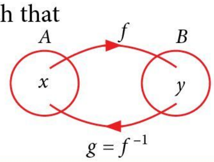
$D_f = R$

$R_f = \{-1, 0, 1\}$



Inverse of a Function

Let : $f : A \longrightarrow B$ is a objective function, then define $g : B \longrightarrow A$ such that $f(x) = y \ \Rightarrow\ g(y) = x$



Even and Odd Function

Even function : $f(-x) = f(x)$

Odd function : $f(-x) = -f(x)$

Arithmetic Progression (A.P.)

A sequence in which each term is obtained by adding a fixed number to the preceding term except the first term.

General form of an A.P.

$$a, a + d, a + 2d, \dots,$$

where a = first term and d = common difference

General term (or n^{th} term) of an A.P.

$$a_n = n^{\text{th}} \text{ term} = a + (n - 1)d$$

n^{th} term from last of an A.P.

$$a'_n = l - (n - 1)d,$$

where n = number of terms

Sum of n terms of an A.P.

$$S_n = \frac{n}{2}[2a + (n - 1)d] \text{ or } S_n = \frac{n}{2}[a + l],$$

where l = last term

Arithmetic mean A.M.

- For two numbers a and b, A.M. is $\dfrac{a+b}{2}$
- $A_r = a + r\left(\dfrac{b-a}{n+1}\right), \forall \ r = 1, 2, \dots, n$

where $A_1, A_2, \dots, A_n$ are n arithmetic means inserted between two numbers a and b.

Geometric Progression (G.P.)

A sequence in which each term is obtained by multiplying the previous term by a non zero constant.

General form of a G.P.

$$a, ar, ar^2, \dots,$$

where a = first term and r = common ratio

Sum of 'n' term of a G.P.

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{or} \quad S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

or $S_n = na, \ r = 1$

Sum of infinite terms of a G.P.

$$S_\infty = \frac{a}{1 - r}; |r| < 1$$

General term (or n^{th} term) of a G.P.

$$a_n = ar^{n-1}$$

n^{th} term from the last of a G.P.

$$a'_n = \frac{l}{r^{n-1}}$$

Geometric mean (G.M.)

- For two numbers a and b, G.M. is $\sqrt{ab}$

$$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}, \forall \ r = 1, 2, \dots, n$$

where $G_1, G_2, \dots, G_n$ are n geometric means inserted between two numbers a and b.

Relation between A.M. and G.M.

- If $a_1, a_2, a_3, \dots, a_n$ are 'n' positive numbers, then A.M. $\geq$ G.M.

i.e., $\dfrac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 \ a_2 \dots a_n)^{\frac{1}{n}}$; Equality holds if $a_1 = a_2 = \dots = a_n$

Permutation

Arrangement of number of things taken some or all of them at a time.

Notation

Number of ways of arranging ‘n’ distinct things taken ‘r’ at a time

i.e., $${}^nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

Properties

- ${}^nP_n = n!$
- ${}^nP_1 = n$
- ${}^nP_0 = 1$
- ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$
- $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1$
- ${}^{n-1}P_r + r\,{}^{n-1}P_{r-1} = {}^nP_r$

Circular Permutation

- Number of circular permutations of n distinct object: $(n - 1)!$
- Number of circular permutation of n distinct objects taken r at a time: ${}^nC_r\,(r - 1)!$

Restricted Permutations

Number of permutations of n objects taken r at a time, when a particular object is :

- Always included (or never excluded) $= r\cdot({}^{n-1}P_{r-1})$
- Always excluded (or never included) $= {}^{n-1}P_r$

Combination

Different groups or selection which can be made by taking some or all the objects.

Notation

Number of selection of ‘n’ distinct things taken ‘r’ at a time, $0 \le r \le n$

i.e., $${}^nC_r = C(n, r) = \binom{n}{r} = \frac{n!}{(r)!\,(n-r)!}$$

Properties

- ${}^nC_r = {}^nC_{n-r}$
- If ${}^nC_x = {}^nC_y$

$\Rightarrow \quad x = y \quad \text{or} \quad x + y = n$

- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Relation between Permutation and Combination

- In permutation, we deal with arrangements of objects whereas in combination, we have to deal with selection of objects.
- Order of objects is not important in combination but has importance in permutation.

Logarithm

Logarithm is the power to which a number must be raised in order to get some other number.

Properties of Logarithm

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x$
- $\log_b a = \log_c a \log_b c$
- $\log_b a = \dfrac{1}{\log_a b}$
- $\log_b a = \dfrac{\log_c a}{\log_c b}$
- $\log_b (xy) = \log_b x + \log_b y$
- $\log_b (x/y) = \log_b x - \log_b y$
- $\log_b a^{\alpha} = \alpha \log_b a$
- $\log_{b^{\beta}} a = \dfrac{1}{\beta} \log_b a$
- $\log_{b^{\beta}} a^{\alpha} = \dfrac{\alpha}{\beta} \log_b a$
- $a^{\log_c b} = b^{\log_c a}$ $(a,\ b,\ c > 0 \text{ and } c \neq 1)$
- $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$
- $\log_2 \log_2 \sqrt{\sqrt{\sqrt{\sqrt{\ldots\sqrt{2}}}}} = -n$, where n is the number of square roots.

Binomial Theorem

The algebraic expansion of powers of a binomial.

Binomial theorem for any positive integer *n*.

$$(x+y)^n = {}^nC_0\, x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_n\, x^0 y^n$$

i.e., $(x+y)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r\, x^{n-r} b^r$,

where ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ are known as binomial coefficients.

- $(x-y)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r (-1)^r x^{n-r} y^r$
- $(1+x)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r x^r$
- $(1-x)^n = \displaystyle\sum_{r=0}^{n} (-1)^r\ {}^nC_r\ x^r$

Properties of Binomial Coefficients

- $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$
- $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
- $C_0 - C_1 + C_2 - \ldots + (-1)^n C_n = 0$
- $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = {}^{2n}C_n$
- $C_0 C_r + C_1 C_{r+1} + C_2 C_{r+2} + \ldots + C_{n-r} C_n = {}^{2n}C_{n-r}$
- $\displaystyle\sum_{r=0}^{n} r\, {}^nC_r = n2^{n-1}$
- $\displaystyle\sum_{r=0}^{n} r(r-1)\, {}^nC_r = n(n-1)\, 2^{n-2}$
- $C_0 + \dfrac{C_1}{2} + \dfrac{C_2}{3} + \ldots\ldots + \dfrac{C_n}{n+1} = \dfrac{2^{n+1}-1}{n+1}$
- $C_0 - \dfrac{C_1}{2} + \dfrac{C_2}{3} - \dfrac{C_3}{4} + \ldots\ldots + \dfrac{(-1)^n C_n}{n+1} = \dfrac{1}{n+1}$

Number of terms

- For index n, $(a+b)^n$ has $(n+1)$ terms.
- $(a+b)^n + (a-b)^n = 2\{{}^nC_0 a^n + {}^nC_2 a^{n-2} b^2 + {}^nC_4 a^{n-4} b^4 + \ldots\}$ and $(a+b)^n - (a-b)^n = 2\{{}^nC_1\, a^{n-1} b^1 + {}^nC_3 a^{n-3} b^3 + {}^nC_5 a^{n-5} b^5 + \ldots\}$
- If n is odd, then number of terms in both $\{(a+b)^n + (a-b)^n\}$ and $\{(a+b)^n - (a-b)^n\} = (n+1)/2$
- If n is even, then number of terms in
  $\{(a+b)^n + (a-b)^n\} = n/2 + 1$; $\{(a+b)^n - (a-b)^n\} = n/2$

Complex Number

Combination of both the real number and imaginary number.

Definition

A number of the form $a + ib$, where $a, b \in R$ and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

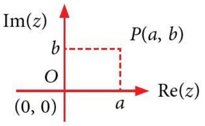
$$z = a + ib$$

Real part

Imaginary part

Argand Plane

$z = a + ib$ is represented by a point $P(a, b)$



Modulus, Conjugate & Argument

Modulus of $z = a + ib$ is

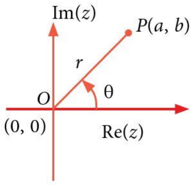
$r = |z| = \sqrt{a^2 + b^2}$

Conjugate of $z = a + ib$ is

$\overline{z} = a - ib$

Argument of $z = a + ib$ is

$$\theta = \tan^{-1}\left(\frac{\text{lm }(Z)}{\text{Re }(Z)}\right), \quad -\pi < 0 \le \pi$$



Cube roots of unity

Let $x = (1)^{1/3} \Rightarrow x^3 - 1 = 0$

$$\Rightarrow x = 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2} \text{ or } x = 1, \omega, \omega^2$$

- $\omega^3 = 1$
- $1 + \omega + \omega^2 = 0$
- $1 + \omega^n + \omega^{2n} = \begin{cases} 0, & \text{if } n \text{ is a not multiple of 3} \\ 3, & \text{if } n \text{ is a not multiple of 3} \end{cases}$

Power of *i*

Integral powers of i

$i^0 = 1, i^1 = i, i^2 = -i, i^3 = i^2(i) = -i, i^4 = (i^2)^2$
$= (-1)^2 = 1$

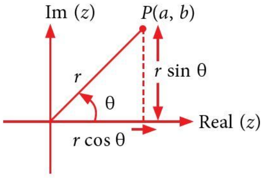
In general, for any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$

Square root of complex number

$$\sqrt{a + ib} = \begin{cases} \pm\left(\sqrt{\dfrac{|z| + a}{2}} + i\sqrt{\dfrac{|z| - a}{2}}\right), & \text{for } b > 0 \\ \pm\left(\sqrt{\dfrac{|z| + a}{2}} - i\sqrt{\dfrac{|z| - a}{2}}\right), & \text{for } b < 0 \end{cases}$$

Polar representation

$Z = r[\cos(2n\pi + \theta) + i \sin(2n\pi + \theta)], n \in I$



Inequalities in complex numbers

If z_1 and z_2 are two complex numbers such that $|z_1| \le 1$, $|z_2| \le 1$, then

- $|z_1 - z_2|^2 \le (|z_1| - |z_2|)^2 + (\text{Arg }(z_1) - \text{Arg}(z_2))^2$
- $|z_1 + z_2|2 \ge (|z_1| + |z_2|)^2 - (\text{Arg }(z_1) - \text{Arg }(z_2))^2$
- $|z_1 \pm z_2| \le |z_1| + |z_2|$
  In general $|z_1 \pm z_2 \pm z_3 \pm \ldots \pm z_n| \le |z_1| + |z_2| + |z_3| + \ldots + |z_n|$
- $|z_1 \pm z_2| \ge ||z_1| - |z_2||$
  $\Rightarrow ||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$

Quadratic Equations

An equation of the form $ax^2 + bx + c, a \neq 0$ where a, b, c are real or complex number.

Common Roots of Quadratic Equation

Condition of one common root

Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root α.

$$\therefore \alpha = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}, \alpha \neq 0$$

Condition for both the common roots

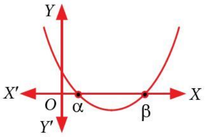
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Quadratic Expression and its Graph

Case-I : For $a > 0$ and $D > 0$

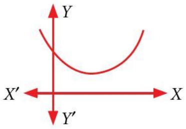
For $\alpha < x < \beta$,

$y < 0$



Case-III : For $a > 0$ and $D < 0$

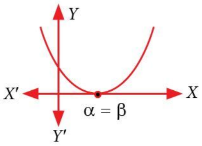
In this case, imaginary roots appears & $y > 0$ for $x \in R$.



Case-II : For $a > 0$ and $D = 0$

$y = a(x - \alpha)^2$

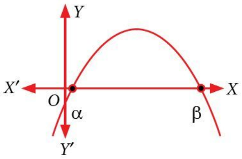
and $y \geq 0$, for $x \in R$.



Case-IV : For $a < 0$ and $D > 0$

$y > 0$, if $\alpha < x < \beta$

$y < 0$, if $x < \alpha$ or $x > \beta$



Matrices

Any rectangular arrangement of numbers.

Types of Matrices

Orthogonal matrix : $A = [a_{ij}]_{m \times n}$

if $AA' = I_n = A'A$, then A is orthogonal matrix.

Idempotent matrix : $A = [a_{ij}]_{n \times n}$

If $A^2 = A$, then A is idempotent matrix.

Involutory matrix : $A = [a_{ij}]_{n \times n}$

If $A^2 = I$, then A is involutory matrix.

Nilpotent matrix : $A = [a_{ij}]$

If a positive integer k such that $A^k = 0$, then A is Nilpotent matrix.

Unitary matrix : $A = [a_{ij}]$

If $A' A = I$, then A is unitary matrix.

Rank of matrix

Rank of matrix = Maximum number of linearly independent columns (or rows of a matrix)

Trace of matrix

If $A = [a_{ij}]_{n \times n}$, then, $tr(A) = \sum_{i=j=1}^{n} a_{ij}$

- $tr(A) + tr(B) = tr(A + B)$
- $tr(kA) = k\, tr(A)$

Order of matrix

A matrix having m rows and n columns is called matrix of order $m \times n$.

Determinants

A compact form showing a set of numbers arranged in same number of rows and columns.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the row and the column of the element a_{ij}, then the value of determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij}.
- The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and denoted by A_{ij}. $\therefore \quad A_{ij} = (-1)^{i+j} M_{ij}$

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- Singular if $|A| = 0$
- Non-singular if $|A| \neq 0$

Properties

- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1} A^{-1}$
- $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

Inverse of a Matrix

For any square matrix A, inverse of A is defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj } A), |A| \neq 0$$

Area of a triangle

Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

$$\Delta = \frac{1}{2}\left\| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right\|$$

Adjoint of a Matrix

Let $B = [A_{ij}]$ be the matrix of cofactors of matrix $A = [a_{ij}]$. Then the transpose of B is called the adjoint of matrix A.

Properties of adj (A)

If A is non-singular matrix of order n, then

- $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$
- $|A \text{ adj } A| = |A|^n$
- $\text{adj } (AB) = (\text{adj } B)\, .\, (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj } (\text{adj } A) = |A|^{n-2} A$
- $|\text{adj } (\text{adj } A)| = |A|^{(n-1)2}$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations :

- If $|A| \neq 0$, the system is consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

Cramer's rule

- If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ then, $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$

  where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$,
- If $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$

  then, $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$, $z = \frac{D_3}{D}$,

  where $D \neq 0$

GK CORNER

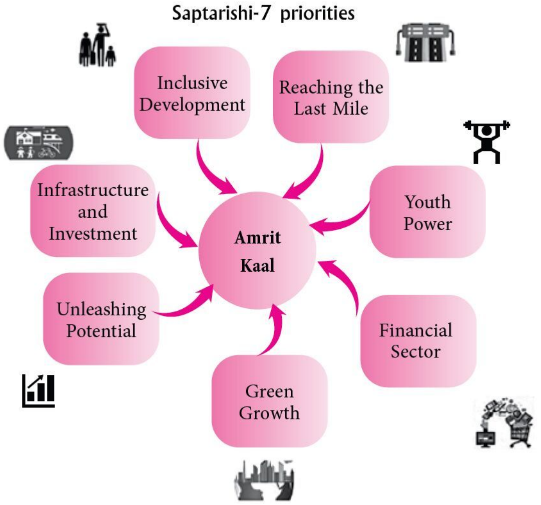
Enhance Your General Knowledge with Current Updates!

INDIAN ECONOMY

Union Budget 2023-24

- The Finance Minister of India introduced the Union Budget (for 2023-24) before the Parliament on February 1, 2023. Highlights of Budget 2023-24 are:
  - Changes in the new income tax regime (in rebate limit and in tax slabs).
  - A 33% increase in capital investment outlay has been proposed, raising it to ₹ 10 lakh crore (the biggest in the past decade).
  - Capital outlay for the railways increased to the highest ever – ₹ 2.40 lakh crore.

Priorities of Budget 2023-24


Saptarishi-7 priorities
Inclusive Development
Reaching the Last Mile
Infrastructure and Investment
Amrit Kaal
Youth Power
Unleashing Potential
Financial Sector
Green Growth

Nidhi Aapke Nikat 2.0 Programme

- Employees' Provident Fund Organisation (EPFO) has launched a massive District Outreach Program in all the districts of the country through a revamped Nidhi Aapke Nikat program. The aim of this program is to reach all the districts of the country on the same day *i.e.* 27th of every month. For that, the EPFO has planned to organize camps in 685 districts of the country.

Complete Transition to 'Trade-Plus-One' (T+1) Settlement Cycle

- India becomes the second country in the world after China to start 'Trade-plus-one' (T+1) settlement cycle in top listed securities.
- T+1 (trade-plus-one) means that trade-related settlements will be cleared within a day of the actual transactions. Earlier, trades on the Indian exchanges were settled in two working days after the transaction took place (T+2).

Financial Stability Report (FSR)

- Recently, the RBI released its 26th FSR which reflects the collective assessment of the Sub-Committee of the Financial Stability and Development Council (FSDC) on risks to financial stability, as also the resilience of the financial system. It is released twice every year.

RBI's e₹

- The RBI launched Central Bank Digital Currency (CBDC) or e-Rupee (e₹) in 2022 for retail markets in a pilot mode. It is in the form of a digital token that represents legal tender. Reliance Retail becomes the first retail to accept payments via e-rupee at stores. Reliance retail has

DIGITAL RUPEE (e₹) ACCEPTED HERE
e₹
DIGITAL RUPEE
ACCEPTED HERE

partnered with ICICI Bank and Kotak Mahindra Bank to enable acceptance of RBI-issued Digital Rupee.

Global Wage Report 2022-23

- The Global Wages Report by the International Labour Organisation (ILO) presents an analysis of how concurrent crises – COVID-19 pandemic followed by cost-of-living crisis – have impacted wages and purchasing power across countries and regions.
- Global wages declined in 2022 for the first time since the 2008 global financial crisis due to increasing living expenses. This is threatening to worsen inequality and trigger social unrest.

Test Yourself!

1. Which of the following banks are categorised as Domestically Systemically Important Banks (D-SIBs) or banks "too big to fail"?
(i) State Bank of India (ii) Punjab National Bank
(iii) HDFC Bank (iv) ICICI Bank
Select the correct option.
(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (i), (iii) and (iv) only (d) (ii), (iii) and (iv) only

2. Which among the following organisation releases Global Wage Report 2022-23?
(a) Bank for International Settlements
(b) World Bank
(c) International Monetary Fund
(d) International Labour Organisation

3. Recently, New T+1 settlement cycle has come into effect. Select the incorrect statement about T+1 Settlement.
(a) It means that trade-related settlements must be done within a day, or 24 hours, of the completion of a transaction.
(b) Until 2001, stock markets had a weekly settlement system.
(c) After China and UK, India will become the third country in the world to start the 'trade-plus-one' (T+1) settlement cycle in top listed securities.
(d) None of the above

4. '*Nidhi Apke Nikat*' is an outreach program conducted by which institution?
(a) Department of Legal Affairs
(b) Employees' Provident Fund Organisation
(c) Department of Border Management
(d) Khadi and Village Industries Commission

5. Recently, the Reserve Bank of India (RBI) released its 26th Financial Stability Report (FSR). It reflects the collective assessment of
(a) Macro-Economic Framework Statement
(b) Sub-Committee of the Financial Stability and Development Council
(c) Annual Financial Statement
(d) Board for Regulation and Supervision of Payment and Settlement Systems

6. Consider the following statements about the budget allocations of fiscal year 2023-24 across the ministries and choose which is the correct answer?
(a) Ministry of Defence has the largest share in budget allocations
(b) Ministry of Railways has largest share in budget allocations
(c) Ministry of Agriculture and Farmer's Welfare has received highest allocation amongst the ministries
(d) None of the above

7. RBI has launched a virtual form of currency under the name Digital Rupee. Who can invest in digital money?
(a) Investors
(b) Secondary market transactions in government securities
(c) Wholesale and retail customers
(d) All of the above

8. Which of the following statements is incorrect about Indian economy?
(a) The capital expenditure of Union government has increased approximately four times since Fiscal year 2016
(b) The non-performing assets of banks have been falling for last 2 years
(c) There has been constant increase in forex reserves.
(d) All of the above

9. According to Union Budget 2023-24, which among the following are included in seven priorities of government?
(i) Reaching the last mile (ii) Green growth
(iii) Reducing inequality (iv) Unleashing potential
(a) Only (i), (ii) and (iii) (b) Only (ii) and (iv)
(c) Only (i), (ii) and (iv) (d) All of the above

10. According to the Union Budget, 2023-24, what is the rebate limit of Personal Income Tax in the new tax regime?
(a) 8 lakh (b) 6 lakh
(c) 9 lakh (d) 7 lakh

Answer Key

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(b)
6.	(a)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(d)

◆ ◆

UCD
Unique Career in Demand

Explore the available Unique Career Options!

B.Sc. in Actuarial Science

Bachelor of science in Actuarial Science course is 3-year undergraduate course, where students learn about insurance risk calculation, insurance premiums, application of economic and mathematical analysis for making any decision in financial planning, insurance investment and management among all the others. This is the best course to learn core mathematical skills.

Higher Studies options after B.Sc in Actuarial Science

- Master of Science in Actuarial Science
- Post Graduate Diploma in Actuarial Science
- PG Diploma in Insurance and Risk Management
- PG Diploma in Insurance Science
- PG Diploma in management of Insurance and Financial Services
- B.Sc. Actuarial Science graduates can also opt for MBA course as it will give them in-depth knowledge about the organizational work structure.

Job Perspectives

Actuaries mostly work in Insurance and Reinsurance companies. They also work as consultants for companies, banking, financial consulting firms, actuarial consulting firms, the government sector, pension funds, investment advisories and healthcare units.

According to the Bureau of labour statistics, the number of actuaries employed is expected to grow 21% from 2021 to 2031. For this reason, many universities offering educational degrees and courses on Actuarial science.

Eligibility Criteria

- The admission for the Actuarial Science course will be based on the merit scored in the Actuarial Common Entrance Exam (ACET) held by the Institute of Actuaries of India (IAI).
- The student must complete an Intermediate degree or 12th final year examination.
- The minimum percentage required may vary but candidates must have scored at least 70% marks in aggregate in the compulsory subjects.

JEE WORK OUTS

Single Option Correct Type

1. $f:\left(0, \frac{\pi}{2}\right)\to A,\ f(x)=\log_e(\sin x^{\sin x}+1),$ then the minimum value of $f(x)$ is

(a) $\log_e 2$ (b) $\log_e\left(\left(\frac{1}{e}\right)^{1/e}+1\right)$

(c) $\log_e((e)^2+1$ (d) 2

2. If $x^5 - x^3 + x = a$, then minimum value of x^6 is

(a) a (b) $2a - 1$ (c) $a + 2$ (d) $a - 3$

3. If z be a complex number satisfying, $|z|^2 + 2(z + \bar{z}) + 3i(z - \bar{z}) + 4 = 0$ then complex number $z + 3 + 2i$ will lie on

(a) circle with centre $1 - 5i$ and radius 4

(b) circle with centre $1 + 5i$ and radius 4

(c) circle with centre $1 + 5i$ and radius 3

(d) circle with centre $1 - 5i$ and radius 3

4. If $f(x - y) = f(x)g(y) - f(y)g(x)\ \forall\ x, y \in R$, where $f(x)$ is not identically zero, then

(a) If $f'(0^+)$ exists then $f'(0^-)$ also exists but not equal

(b) If $f'(0^+)$ exists then $f'(0^-)$ does not exist

(c) If $f'(0^+)$ exists then it is equal to $f'(0^-)$

(d) none of these

5. A quadratic polynomial maps from $[-2, 3]$ onto $[0, 3]$ and touches x-axis at $x = 3$, then the polynomial is

(a) $\frac{3}{16}(x^2-6x+16)$ (b) $\frac{3}{25}(x^2-6x+9)$

(c) $\frac{3}{25}(x^2-6x+16)$ (d) None of these

6. Let $f(x)=\int_0^x (t^2+2t+2)dt,$ where x belongs to the set of real numbers satisfying the inequation $\log_2(1+\sqrt{6x-x^2-8})\geq 0.$ If $f(x) \in [\alpha, \beta]$, then maximum value of $|\alpha - \beta|$ is

(a) 103/6 (b) 104/3

(c) 106/3 (d) None of these

7. The set of values of a for which inequation $(a - 1)x^2 - (a + 1)x + a - 1 \geq 0$ is true for all $x \geq 2$ is

(a) $\left(1, \frac{7}{3}\right]$ (b) $(-\infty, 1)$

(c) $\left[\frac{7}{3}, \infty\right)$ (d) None of these

8. If $x_1, x_2, x_3, ..., x_n$ are the roots of $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)...(x_1 - x_n)$ is

(a) n_1^n (b) $nx_1^{n-1} + a$

(c) nx_1 (d) $nx_1 + a^n$

9. The value of $\lim\limits_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

(a) em (b) e/m

(c) $1/em$ (d) None of these

10. If one of the roots of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}=0$ is $x = 2$, then sum of all other five roots is

(a) $2\sqrt{15}$ (b) -2

(c) $\sqrt{20}+\sqrt{15}-2$ (d) None of these

11. A unit vector is orthogonal to $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar to $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$, then the vector is

(a) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (b) $\frac{2\hat{i}+5\hat{j}}{\sqrt{29}}$

(c) $\dfrac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (d) $\dfrac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

12. The range of the function $y=\sqrt{2\{x\}-\{x\}^2-\dfrac{3}{4}}$ is (where $\{\cdot\}$ denotes fractional part)

(a) $\left[-\dfrac{1}{4},\dfrac{1}{4}\right]$ (b) $\left[0,\dfrac{1}{2}\right)$

(c) $\left[0,\dfrac{1}{4}\right]$ (d) $\left[\dfrac{1}{4},\dfrac{1}{2}\right]$

13. $\displaystyle\int\frac{dx}{\sqrt{\cos^3 x\cos(x-\alpha)}}$ is equal to

(a) $2\sqrt{\tan x\sin\alpha+\cot\alpha}+c$

(b) $2\operatorname{cosec}\alpha\sqrt{\tan x\sin\alpha+\cos\alpha}+c$

(c) $2\sqrt{\sin\alpha+\tan x\cos\alpha}+c$

(d) $2\operatorname{cosec}\alpha\sqrt{\tan x\cos\alpha+\sin\alpha}+c$

14. Inside a big circle exactly n small circles each of radius r can be drawn in such a way that each small circle touches the big circle and two small circles. If $n \geq 3$, then the radius of the bigger circle is

(a) $r\operatorname{cosec}\left(\dfrac{\pi}{n}\right)$ (b) $r\left\{1+\operatorname{cosec}\left(\dfrac{\pi}{n}\right)\right\}$

(c) $r\left\{1+\operatorname{cosec}\left(\dfrac{2\pi}{n}\right)\right\}$ (d) $r\left\{1+\operatorname{cosec}\left(\dfrac{\pi}{2n}\right)\right\}$

15. If m_1 and m_2 be the slopes of two perpendicular chords of equal length passing through origin of circle $(x-1)^2+(y+2)^2=5$, then the value of $m_1^2+m_2^2$ is equal to

(a) 80/9   (b) 82/9

(c) 83/9   (d) None of these

One or More Than One Option(s) Correct Type

16. Let $f:[a,b]\to[1,\infty)$ be a continuous function and let $g:R\to R$ be defined as

$$g(x)=\begin{cases}0, & \text{if } x<a\\ \displaystyle\int_a^x f(t)\,dt, & \text{if } a\leq x\leq b\\ \displaystyle\int_a^b f(t)\,dt, & \text{if } x>b\end{cases}$$

Then

(a) $g(x)$ is continuous but not differentiable at a

(b) $g(x)$ is differentiable on R

(c) $g(x)$ is continuous but not differentiable at b

(d) $g(x)$ is continuous and differentiable at either a or b but not both

17. Let M and N be two 3×3 matrices such that $MN=NM$. Further, if $M\neq N^2$ and $M^2=N^4$, then

(a) determinant of (M^2+MN^2) is 0

(b) there is a 3×3 non-zero matrix U such that $(M^2+MN^2)U$ is the zero matrix

(c) determinant of $(M^2+MN^2)\geq1$

(d) for a 3×3 matrix U, if $(M^2+MN^2)U$ equals the zero matrix then U is the zero matrix.

18. An ellipse intersects the hyperbola $2x^2-2y^2=1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(a) Equation of ellipse is $x^2+2y^2=2$

(b) The foci of ellipse are $(\pm1, 0)$

(c) Equation of ellipse is $x^2+2y^2=4$

(d) The foci of ellipse are $(\pm\sqrt{2}, 0)$

19. If $I_n=\displaystyle\int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}\,dx,\ n=0,1,2,\ldots$, then

(a) $I_n=I_{n+2}$ (b) $\displaystyle\sum_{m=1}^{10}I_{2m+1}=10\pi$

(c) $\displaystyle\sum_{m=1}^{10}I_{2m}=0$ (d) $I_n=I_{n+1}$

20. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\pi/3$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(a) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$ (b) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$

(c) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$ (d) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

21. Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If Arg(ω) denotes the principal argument of a non-zero complex number ω, then

(a) $|z-z_1|+|z-z_2|=|z_1-z_2|$

(b) $\operatorname{Arg}(z-z_1)=\operatorname{Arg}(z-z_2)$

(c) $\begin{vmatrix}z-z_1 & \bar{z}-\bar{z}_1\\ z_2-z_1 & \bar{z}_2-\bar{z}_1\end{vmatrix}=0$

(d) $\operatorname{Arg}(z-z_1)=\operatorname{Arg}(z_2-z_1)$

22. If $\sin y=x\cos(a+y)$, then dy/dx is

(a) $\dfrac{\cos^2(a+y)}{\cos a}$ (b) $\dfrac{\cos(a+y)}{x\sin(a+y)+\cos y}$

(c) $\dfrac{\cos a}{3x^2-2x\cos a-1}$ (d) none of these

23. The det $\Delta = \begin{vmatrix} d^2+r & de & df \\ de & e^2+r & ef \\ df & ef & f^2+r \end{vmatrix}$ is divisible by

(a) r (b) $(d + e^2 + f^2 + r)$
(c) $(d^2 + e^2 + f^2 + r)$ (d) $(d^2 + e + f^2 + r^2)$

24. If both the roots of equation $ax^2 + x + c - a = 0$ are imaginary and $c > -1$, then

(a) $3a > 2 + 4c$ (b) $a > 0$
(c) $c < a$ (d) None of these

25. The solution of the equation

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28 \text{ is}$$

(a) $(2n+1)\frac{\pi}{2}, n\in I$ (b) $n\pi-\frac{\pi}{4}, n\in I$
(c) $2n\pi$ (d) none of these

Numerical Answer Type

26. Let m and n be two positive integers greater than 1. If $\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$, then the value of $\frac{m}{n}$ is ______.

27. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ______.

28. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0,0)$, $Q(1,1)$ and $R(2,0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ $(n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ______ .

29. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is ______.

30. In ΔABC if BC is unity,

$\sin\frac{A}{2}=x_1$, $\sin\frac{B}{2}=x_2$, $\cos\frac{A}{2}=x_3$ and $\cos\frac{B}{2}=x_4$ with $\left(\frac{x_1}{x_2}\right)^{2007}-\left(\frac{x_3}{x_4}\right)^{2006}=0$, then the length of AC is ________.

SOLUTIONS

1. (b): Let $h(x) = \log(x^x + 1)$, for $0 < x < 1$. Then, $h'(x) > 0$ if $x > \frac{1}{e}$. Thus, for $\frac{1}{e} < x < 1$, function will be increasing. Also, $h'(x) < 0$, if $x < \frac{1}{e}$. So, at $x = \frac{1}{e}$, there is minimum. $f_{\min} = \log_e\left(\left(\frac{1}{e}\right)^{1/e}+1\right)$.

2. (b): $x(x^4-x^2+1)=\frac{x(x^6+1)}{x^2+1}$. Suppose $x > 0$, then $a>0$ and $x^6+1=\frac{a(x^2+1)}{x}=a\left(x+\frac{1}{x}\right)\ge 2a, x^6>2a-1.$

If $x \le 0 \Rightarrow a \le 0, 2a - 1 < 0 \le x^6 \Rightarrow x^6 > 2a - 1.$

3. (c): Given equation is
$|z|^2 + z(2 + 3i) + \overline{z}(2 - 3i) + 4 = 0$, which represents a circle with centre $-(2 - 3i)$, radius $= \sqrt{a\overline{a}-b}=3$
Let $\omega = z + 3 + 2i = z + 2 - 3i + 1 + 5i$,
$|\omega - 1 - 5i| = |z + 2 - 3i| = 3$
so, ω lies on circle whose centre is $1 + 5i$, radius = 3.

4. (c): $f(x - y) = f(x)g(y) - f(y)g(x)$ ...(i)
Put $x = y$ in (i), $f(0) = 0$,
Put $y = 0$ in (i), $f(x) = f(x)\, g(0) - f(0)\, g(x) \Rightarrow g(0) = 1$

Now, $f'(0^+)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{f(0)g(-h)-f(-h)g(0)}{h}=\lim_{h\to 0}\frac{f(-h)}{-h}=f'(0^-)$

i.e. $f'(0^+)=f'(0^-)$.

5. (b)

6. (b): $\log_2(1+\sqrt{6x-x^2-8})\ge 0$

$\Rightarrow 1+\sqrt{6x-x^2-8}\ge 1$

$\Rightarrow 6x-x^2-8\ge 0 \Rightarrow x^2-6x+8\le 0$

$\Rightarrow (x - 2)(x - 4) \le 0 \Rightarrow 2 \le x \le 4$

Now $f'(x) = x^2 + 2x + 2 > 0\ \forall\ x \in R$

$\Rightarrow f(x)$ is strictly increasing in [2, 4]

$f(x)=\frac{x^3}{3}+x^2+2x, \alpha=f(2)=\frac{8}{3}+4+4=\frac{32}{3}$

$\beta=f(4)=\frac{64}{3}+16+8=\frac{136}{3}, |\alpha-\beta|_{\max}=\frac{104}{3}$

7. (c)

8. (b): Given $x^n + ax + b = 0 = (x - x_1)(x - x_2)(x - x_3) \ldots (x - x_n)$

$\Rightarrow (x-x_2)(x-x_3)\ldots(x-x_n)=\frac{x^n+ax+b}{x-x_1}$

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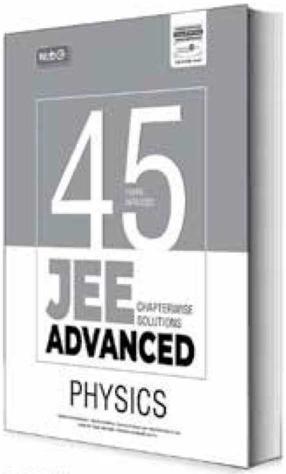
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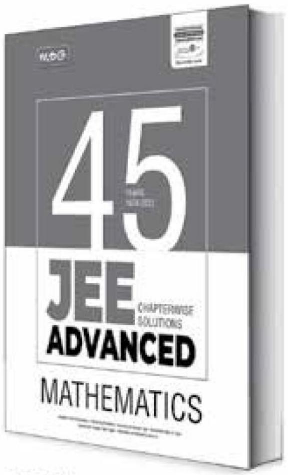
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$\therefore \quad (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \ldots (x_1 - x_n)$

$$= \lim_{x\to x_1}\left(\frac{x^n+ax+b}{x-x_1}\right) = nx_1^{n-1}+a.$$

9. (c)

10. (b): Let $x^2 - 13 = t$, then $t^3 - 67t + 126 = 0$ $\Rightarrow t=-9, 2, 7 \Rightarrow x=\pm 2, \pm\sqrt{20}, \pm\sqrt{15}$.

11. (a): A vector orthogonal to $\vec{c}$ and coplanar with $\vec{a}$ and $\vec{b}$ is $\vec{c}\times(\vec{a}\times\vec{b})$

Here, $\vec{c}=(5,2,6), \vec{a}=(2,1,1), \vec{b}=(1,-1,1)$

Then $\vec{c}\times(\vec{a}\times\vec{b})=(0, 27, -9)$

$\therefore$ Unit vector is $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$.

12. (b): Set $\{x\} = t$ so that $0 \le t < 1$, $y=\sqrt{-t^2+2t-\frac{3}{4}}$

The domain of function is $\frac{1}{2} \le t < 1$.

$$y\left(\frac{1}{2}\right)=\sqrt{-\frac{1}{4}+1-\frac{3}{4}}=0\,;\; y(1)=\sqrt{-1+2-\frac{3}{4}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

But $\frac{1}{2}$ is not attained, so the range is $\left[0, \frac{1}{2}\right)$.

13. (b): Let $I = \displaystyle\int \frac{dx}{\sqrt{\cos^3 x(\cos x\cos\alpha+\sin x\sin\alpha)}}$

$= \displaystyle\int \frac{\sec^2 x dx}{\sqrt{\cos\alpha+\tan x\sin\alpha}}$, Put $\tan x\sin\alpha+\cos\alpha=t^2$

$\Rightarrow \sec^2 x dx = \dfrac{2tdt}{\sin\alpha} \quad \therefore I = \displaystyle\int\frac{2tdt}{\sin\alpha\cdot t} = 2\operatorname{cosec}\alpha\cdot t$

$= 2\operatorname{cosec}\alpha\sqrt{\tan x\sin\alpha+\cos\alpha}+c$

14. (b): We have, $\sin\left(\frac{\pi}{n}\right)=\frac{r}{R-r} \Rightarrow R-r=r\operatorname{cosec}\left(\frac{\pi}{n}\right)$

$\therefore \; R=r\left\{1+\operatorname{cosec}\left(\frac{\pi}{n}\right)\right\}$

15. (b): Let slope of chord, $OA = m$, $\tan 45^\circ=\left|\frac{m+2}{1-2m}\right|$

$\Rightarrow |m+2|=|1-2m| \Rightarrow m+2=\pm(1-2m) \Rightarrow m=3$ or $-\frac{1}{3}$

$\Rightarrow m_1^2+m_2^2=(m_1+m_2)^2-2m_1m_2=\frac{64}{9}+2=\frac{82}{9}$

16. (a, c) : $g(a^-) = 0$

Also, $g(a^+)=\displaystyle\lim_{h\to 0}\int_a^{a+h} f(t)dt=0$

Thus g is continuous at $x = a$. Similarly, g is continuous at $x = b$.

$$g'(a^-)=\lim_{h\to 0}\frac{g(a-h)-g(a)}{-h} \text{ (by definition)}$$

$$=\lim_{h\to 0}\frac{0-\int_a^a f(t)dt}{-h}=\lim_{h\to 0}\frac{0}{-h}=0$$

$$g'(a^+)=\lim_{h\to 0}\frac{g(a+h)-g(a)}{h}=\lim_{h\to 0}\frac{\int_a^{a+h} f(t)dt-0}{h}$$

$$=\lim_{h\to 0}\frac{\int_a^{a+h} f(t)dt}{h}=\lim_{h\to 0}\frac{f(a+h)}{1}=f(a)\ge 1$$

So, $g'_{(a^-)} \ne g'_{(a^+)}$

Hence g is not differentiable at $x = a$.

Similarly, g is not differentiable at $x = b$.

17. (a, b) : $M^2 = N^4 \Rightarrow (M - N^2)(M + N^2) = 0$ as M and N commute.

$M - N^2 \ne 0$ so $\det(M + N^2) = 0$

Recall that if $AB = O$ and $A \ne 0$, then $\det B = 0$

Now, $\det(M^2 + MN^2) = \det(M(M + N^2))$
$= (\det M)(\det (M + N^2))$
$= (\det M) \cdot 0 = 0$

Recall that if $\det A = 0$, then $\exists\, X \ne 0$ such that $AX = 0$

Let $M^2 + MN^2$ play the role of A and X that of U, we get $(M^2 + MN^2)U = O$ for some 3×3 non-zero matrix U.

18. (a, b) : The hyperbola is $\dfrac{x^2}{1/2}-\dfrac{y^2}{1/2}=1$ ...(i)

Eccentricity of the hyperbola $= \sqrt{2}$

$\therefore$ Eccentricity of the ellipse $= \dfrac{1}{\sqrt{2}}$

Let the equation of ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\,(a>b)$

Then $a^2e^2 = a^2 - b^2$

$\Rightarrow a^2\cdot\frac{1}{2}=a^2-b^2 \Rightarrow b^2=\frac{a^2}{2} \quad \therefore a^2=2b^2$

The equation is $\dfrac{x^2}{2b^2}+\dfrac{y^2}{b^2}=1$

$x^2 + 2y^2 = 2b^2$ .....(ii)

Let (x_0, y_0) be a point of intersection of hyperbola (i) and ellipse (ii),

Then for hyperbola $\left[\dfrac{dy}{dx}\right]_{(x_0,\,y_0)}=\dfrac{x_0}{y_0}$

and for ellipse $\left[\dfrac{dy}{dx}\right]_{(x_0,\,y_0)}=-\dfrac{x_0}{2y_0}$

As the intersection is orthogonal, we have

$$-\frac{x_0^2}{2y_0^2} = -1 \Rightarrow x_0^2 = 2y_0^2$$

From (i) and (ii), $2x_0^2 - 2y_0^2 = 1$ and $x_0^2 + 2y_0^2 = 2b^2$

Multiplying the 1st equation by $2b^2$ and subtracting we get $4b^2x_0^2 - 4y_0^2b^2 = x_0^2 + 2y_0^2$

$\Rightarrow \quad x_0^2(4b^2 - 1) = 2y_0^2(1 + 2b^2)$

Also $x_0^2 = 2y_0^2$ which gives

$$4b^2 - 1 = 1 + 2b^2 \Rightarrow 2b^2 = 2$$

$\therefore \quad b^2 = 1$

Thus the equation to ellipse is $x^2 + 2y^2 = 2$

$\therefore$ Foci of the ellipse are $\left(\pm\sqrt{a^2 - b^2}, 0\right)$ *i.e.* $(\pm 1, 0)$

19. (a, b, c) : $I_n = \displaystyle\int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx \qquad \ldots(\text{i})$

$$\Rightarrow I_n = \int_{-\pi}^{\pi} \frac{\sin(-nx)}{(1+\pi^{-x})(-\sin x)}dx \qquad \ldots(\text{ii})$$

Adding (i) and (ii), we get

$$2I_n = \int_{-\pi}^{\pi}\left(\frac{1}{1+\pi^x} + \frac{1}{1+\pi^{-x}}\right)\frac{\sin nx}{\sin x}dx = \int_{-\pi}^{\pi}\frac{\sin nx}{\sin x}dx$$

$$\Rightarrow \quad I_n = \frac{1}{2}\int_{-\pi}^{\pi}\frac{\sin nx}{\sin x}dx$$

Now, $I_{n+2} - I_n = \dfrac{1}{2}\displaystyle\int_{-\pi}^{\pi}\frac{\sin(n+2)x - \sin nx}{\sin x}dx$

$$= \frac{1}{2}\int_{-\pi}^{\pi}\frac{2\sin x\cos(n+1)x}{\sin x}dx$$

$$= \int_{-\pi}^{\pi}\cos(n+1)x\,dx = \left(\frac{\sin(n+1)x}{(n+1)}\right)_{-\pi}^{\pi} = 0$$

$\therefore \quad I_{n+2} = I_n$

Thus $I_0 = I_2 = I_4 = \ldots\ldots = I_{10}$

$I_1 = I_3 = I_5 = \ldots\ldots = I_{21}$

$$I_0 = \frac{1}{2}\int_{-\pi}^{\pi} 0\,dx = 0$$

$$\therefore \quad I_1 = \frac{1}{2}\int_{-\pi}^{\pi}\frac{\sin x}{\sin x}dx = \frac{1}{2}\cdot 2\pi = \pi$$

$$\sum_{m=1}^{10} I_{2m} = 0 \text{ and } \sum_{m=1}^{10} I_{2m+1} = 10\pi$$

20. (a, b, c) : $\vec{a} = p(\vec{x}\times(\vec{y}\times\vec{z}))$, where $p \in R - \{0\}$

$$= p\{(\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}\} = p\{\vec{y} - \vec{z}\}$$

$$\left(\because \vec{x}\cdot\vec{y} = |x||y|\cos\frac{\pi}{3} = \sqrt{2}.\sqrt{2}\left(\frac{1}{2}\right) = 1 = \vec{y}\cdot\vec{z} = \vec{z}\cdot\vec{x}\right)$$

Now, $\vec{a}\cdot\vec{y} = p\{\vec{y}\cdot\vec{y} - \vec{y}\cdot\vec{z}\} = p\{2-1\} = p$

$\therefore \quad p = \vec{a}\cdot\vec{y} \qquad (\because \vec{y}\cdot\vec{y} = \vec{y}^2 = (\sqrt{2})^2 = 2)$

Thus, $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y} - \vec{z}) \qquad \ldots\ldots (\text{i})$

Similarly, $\vec{b} = q(\vec{y}\times(\vec{z}\times\vec{x})) = q\{(\vec{y}\cdot\vec{x})\vec{z} - (\vec{y}\cdot\vec{z})\vec{x}\}$

$= q(\vec{z} - \vec{x})$

Now, $\vec{b}\cdot\vec{z} = q(\vec{z}\cdot\vec{z} - \vec{z}\cdot\vec{x}) = q(\vec{z}^2 - \vec{z}\cdot\vec{x}) = q$

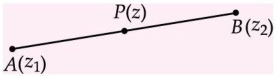
Thus $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z} - \vec{x}) \qquad \ldots\ldots(\text{ii})$

Now, $\vec{a}\cdot\vec{b} = (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})\{(\vec{y} - \vec{z})\cdot(\vec{z} - \vec{x})\}$ (From (i) & (ii))

$$= (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})(\vec{y}\cdot\vec{z} - \vec{y}\cdot\vec{x} - \vec{z}\cdot\vec{z} + \vec{z}\cdot\vec{x})$$

$$= (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})(1-1-2+1) = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$$

21. (a, c, d) : The given statement implies that z is on the line segment joining A and B.



As $z = (1 - t)z_1 + tz_2$ indicate.

$\therefore \quad AP + PB = AB \Rightarrow |z - z_1| + |z - z_2| = |z_1 - z_2|$

Now, z_1, z, z_2 are collinear, so $\text{Arg}\left(\dfrac{z - z_1}{z_2 - z_1}\right) = 0$

giving $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

Note that $\text{Arg}(z - z_1) - \text{Arg}(z - z_2) = \pi$

Form IV

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For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

Also $\dfrac{z-z_1}{\overline{z}-\overline{z}_1}=\dfrac{z_2-z_1}{\overline{z}_2-\overline{z}_1}=$ complex slope of the line which can be rearranged as

$$(z-z_1)(\overline{z}_2-\overline{z}_1)-(z_2-z_1)(\overline{z}-\overline{z}_1)=0$$

i.e., $\begin{vmatrix} z-z_1 & \overline{z}-\overline{z}_1 \\ z_2-z_1 & \overline{z}_2-\overline{z}_1 \end{vmatrix}=0$

22. (a, b) : $\sin y=x\cos(a+y)\Rightarrow\dfrac{\sin y}{\cos(a+y)}=x$ ...(i)

$$\frac{dx}{dy}=\frac{\cos y\cos(a+y)+\sin y\sin(a+y)}{\cos^2(a+y)}=\frac{\cos a}{\cos^2(a+y)}$$

$$\Rightarrow\quad \frac{dy}{dx}=\frac{\cos^2(a+y)}{\cos a} \qquad \text{...(ii)}$$

$$=\frac{\cos^2(a+y)}{\cos y\cos(a+y)+\sin y\sin(a+y)}=\frac{\cos(a+y)}{\cos y+x\sin(a+y)}$$

[Using (i)]

23. (a, c) : Multiply R_1, R_2, R_3 by d, e, f and take out d, e, f common from C_1, C_2, C_3, then

$$\Delta=\begin{vmatrix} d^2+r & d^2 & d^2 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}$$

Operate $R_1\to R_1+R_2+R_3$ and take out $d^2+e^2+f^2+r$ common from R_1, then

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}(d^2+e^2+f^2+r)$$

$$=r^2(d^2+e^2+f^2+r).$$

24. (b) : Here $f(1)=a+1+c-a$

As $c>-1 \quad\Rightarrow\quad f(1)>0$

25. (a, b) : We have,

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28,\ y+\frac{27}{y}=28,$$

$\Rightarrow\quad y^2-28y+27=0\Rightarrow y=1$

$(\because\ y\neq 27$, since max. $(\sin 2x+\cos 2x)=\sqrt{2}\,)$

$\therefore\quad \sin 2x+2\cos^2 x=0\Rightarrow 2\cos x(\sin x+\cos x)=0$

$$\Rightarrow\quad x=(2n+1)\frac{\pi}{2}\text{ or }x=n\pi-\frac{\pi}{4}$$

26. (2): $\displaystyle\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=\lim_{\alpha\to 0}\frac{e(e^{\cos(\alpha^n)-1}-1)}{\alpha^m}$

$$=\lim_{\alpha\to 0}\frac{e(\alpha^n)^2}{\alpha^m}\left\{\frac{e^{\cos(\alpha^n)-1}-1}{\cos(\alpha^n)-1}\right\}\cdot\left\{\frac{\cos(\alpha^n)-1}{(\alpha^n)^2}\right\}=\frac{-1}{2}e\alpha^{2n-m}$$

As limit is given to be $-e/2$, we have

$2n-m=0. \quad\therefore\ m/n=2.$

27. (3748) : Here X = {1, 6, 11, ..., 10086}

and $Y=\{9, 16, 23, ..., 14128\}$

The intersection of X and Y is an A.P. with 16 as first term and 35 as common difference.

The series becomes 16, 51, 86, ...

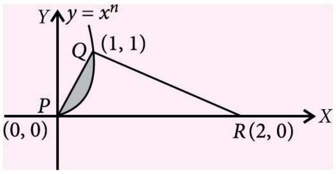
Now, k^{th} term $=16+(k-1)\,35\leq 10086$

i.e. $k\leq\dfrac{10105}{35}\quad\therefore\ k\leq 288$ (as k is to be an integer)

Hence, $n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$

$=2018+2018-288=3748$

28. (4):



$$\int_0^1(x-x^n)dx=\frac{3}{10}\cdot\left(\frac{1}{2}\times 2\times 1\right)$$

$$\Rightarrow\left[\frac{x^2}{2}-\frac{x^{n+1}}{n+1}\right]_0^1=\frac{3}{10}\quad\Rightarrow\quad\frac{1}{2}-\frac{1}{n+1}=\frac{3}{10}$$

$$\Rightarrow\quad\frac{1}{2}-\frac{3}{10}=\frac{1}{n+1}\quad\Rightarrow\quad\frac{2}{10}=\frac{1}{n+1}$$

$\Rightarrow\quad n+1=5\quad\therefore\ n=4$

29. (380) : The team can have either 0 boy or 1 boy. So the number of selection is $({}^6C_4\cdot{}^4C_0+{}^6C_3\cdot{}^4C_1)\cdot{}^4C_1$

$=(15+20\cdot 4)\cdot 4=95\cdot 4=380$

30. (1): In given ΔABC both $A/2$ and $B/2$ lie strictly between $(0, \pi/2)$ and $\sin x$ is always increasing in $(0, \pi/2)$whereas $\cos x$ is always decreasing throughout $(0, \pi/2)$.

So if $\dfrac{A}{2}>\dfrac{B}{2}\ \Rightarrow\sin\dfrac{A}{2}>\sin\dfrac{B}{2}\Rightarrow x_1>x_2$

and $\dfrac{1}{x_3}>\dfrac{1}{x_4}$. Now, $x_1^{2007}x_4^{2006}=x_2^{2007}x_3^{2006}$ is not valid.

Similarly for $A/2<B/2$

$\sin\dfrac{A}{2}<\sin\dfrac{B}{2}\ \Rightarrow x_1<x_2$ and $\dfrac{1}{x_3}<\dfrac{1}{x_4}$. Again equality is not possible.

Therefore only possible case is when $A/2=B/2$

$$\Rightarrow\quad x_1=x_2\text{ and }\frac{1}{x_3}=\frac{1}{x_4}.$$

Hence in this case ΔABC is isosceles with $\angle ABC=\angle CAB$

◆◆

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. $(1+2)(1+2^2)(1+2^4)(1+2^8)(1+2^{16})(1+2^{32})(1+2^{64})(1+2^{128}) = ?$

(a) $2^{512} - 1$ (b) $2^{1024} - 1$
(c) $2^{255} - 1$ (d) $2^{256} - 1$

2. A college has raised 75% of the amount it needs for a new building by receiving an average donation of ₹ 600 from the people already solicited. The people already solicited represent 60% of the people the college will ask for donations. If the college is to raise exactly 25% of the amount needed for the new building, what should be the average donation from the remaining people to be solicited?

(a) ₹ 300 (b) ₹ 250
(c) ₹ 400 (d) ₹ 500

3. Mr. '*X*' invested certain amounts in two different schemes '*A*' & '*B*' for two years. Scheme '*A*' offers simple interest at 12 % p.a. and Scheme '*B*' offers compound interest at 10 % per annum. Interest accrued on the amount invested in Scheme '*A*' in 2 years was ₹ 3,600 and the total amount invested was ₹ 35,000. What was interest accrued on the amount invested in Scheme '*B*' ?

(a) ₹ 4,800 (b) ₹ 4,200
(c) ₹ 4,000 (d) Cannot be determined

4. Given a number 12345678901234567890..... upto 500 digits. Find the smallest number *n* that should be added to the number such that the sum is exactly divisible by 11.

(a) 6 (b) 5 (c) 8 (d) 1

5. A certain manufacture sells a product to the distributor at 10% profit. Then distributor sells it to the dealer and the dealer sells to the retailer at a mark up of 10% and 20% respectively. The retailer marks up his cost by 20% and then offers a 10% discount to the customers. If the customer had bought it from distributor directly, then how much reduction in price would he get with respect to buying it from the retailer?

(a) 29.6% (b) 20% (c) 33.33% (d) 22.8%

6. Mr. *X* invested a certain amount in Debit and Equity funds in the ratio of 4 : 5 respectively. At the end of one year, he earned a total dividend of 30% on his investment. After one year he reinvested the amount including dividend in the ratio of 6 : 7 in Debit and Equity Funds. If the amount reinvested in Equity Funds was ₹ 94,500, what was the original amount invested in Equity Funds?

(a) ₹ 75,000 (b) ₹ 81,0007
(c) ₹ 60,000 (d) ₹ 65,0007

7. Professor John noticed something peculiar while entering the quiz marks of his five students into a spreadsheet. The after each score was entered, Professor John entered the marks in a random order and noticed that after each mark was entered, the average was always an integer. In ascending order, the marks of the students were 71, 76, 80, 82 and 91. What were the fourth and fifth marks that Professor John entered?

(a) 71 and 82 (b) 71 and 76
(c) 71 and 80 (d) 76 and 80

8. The diluted wine contains only 6 litres of wine and the rest is water. A new mixture whose concentration is 50% is to be formed by replacing with wine. How many litres of mixture be replaced with pure wine if there are initially 18 litres of water in the mixture?

(a) 5 litres (b) 6 litres
(c) 8 litres (d) 12 litres

9. If 15 men or 24 women or 36 boys can do a piece of work in 12 days, working 8 hours a day, how many men must be associated with 12 women and 6 boys to do another piece of work $2\frac{1}{4}$ times as great in 30 days working 6 hours a day?

(a) 4 (b) 8 (c) 6 (d) 10

10. Navjivan Express from Ahmedabad to Chennai leaves Ahmedabad at 6:30 a.m. and travels at 50 kmph towards Baroda situated 100 km away. At 7:00 a.m.

Howrah-Ahmedabad Express leaves Baroda towards Ahmedabad and travels at 40 kmph. At 7:30 a.m., Mr. Shah, the traffic controller at Baroda realizes that both the trains are running on the same track. How much time does he have to avert a head-on collision between the two trains?

(a) 20 minutes (b) 30 minutes
(c) 45 minutes (d) None of these

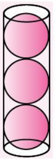
11. The 288th term of the series *a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, f, f, f, f, f, f…* is

(a) *u* (b) *v* (c) *w* (d) *x*

12. Rahim sells two types of coffees viz. DC and VC. He sells DC at ₹ 18 per kg and incurs a loss of 10% whereas on selling the VC at ₹ 30 per kg, he gains 20%. In what proportion should the DC and VC be mixed such that he can gain a profit of 25% by selling the mixture at ₹ 27.5 per kg?

(a) 5 : 3 (b) 3 : 2 (c) 2 : 3 (d) 7 : 4

13. Three identical balls fit snugly into a cylindrical can. The radius of each of the spheres equals the radius of the can and the balls just touch the bottom and the top of the can. What fraction of the volume of the can is taken up by the balls?



(a) 3/5 (b) 4/3 (c) 3/2 (d) 2/3

14. How many four letters words can be formed using the letters of 'SERIES'?

(a) 24 (b) 42 (c) 84 (d) 102

15. From amongst the available 15 cricket players, of whom 5 are bowlers, a team of 11 is to be selected. What is the probability that the selected team will have at least 3 bowlers?

(a) 7/13 (b) 5/13 (c) 12/13 (d) 9/13

16. A water tank has three taps *A, B* and *C*. *A* fills 8 glasses in 48 minutes, *B* fills 16 glasses in 2 hour, and *C* fills 4 glasses in 40 minutes. If all the taps are opened together, a full tank is emptied in 4 hours. If a glass contains 500 ml water, what is the capacity of the tank?

(a) 32 litres (b) 48 litres (c) 56 litres (d) 64 litres

17. The speed of the boat in still water is 16 kmph and speed of the current is 2 kmph. It takes a total of 6.5 hours to row upstream from point *A* to point *B* and downstream from point *B* to point *C*. If the distance from point *A* to point *B* is two-third the distance between point *B* and *C*, what is the total distance travelled by the boat (both upstream and downstream)?

(a) 112 km (b) 98 km (c) 124 km (d) 105 km

18. Left pan of a faulty balance weighs 100 gram more than its right pan. A shopkeeper keeps the weight measure in the left pan while buying goods but keeps it in the right pan while selling his goods. He uses only 1 kg weight measure. If he sells his goods at the listed cost price, what is his gain?

(a) $\frac{200}{9}\%$ (b) $\frac{100}{11}\%$ (c) $\frac{100}{9}\%$ (d) $\frac{200}{11}\%$

19. ₹ 1500 were invested for 5 years in scheme *A* which offers simple interest at the rate of 14% per annum. The amount received after 5 years and some additional money, is then invested in scheme *B*, for 2 years, which offers compound interest (compounded annually) at the rate of 20% per annum. If the compound interest received from scheme *B* after 2 years is ₹ 1408, what was the additional money invested in scheme *B* apart from the amount received from scheme *A*?

(a) ₹ 450 (b) ₹650 (c) ₹ 500 (d) ₹ 280

20. In 2013, the arithmetic mean of the annual incomes of Ram and Shyam was ₹ 3800. The arithmetic mean of the annual incomes of Shyam and Pratap was ₹ 4800, and the arithmetic mean of the annual incomes of Pratap and Ram was ₹ 5800. What is the arithmetic mean of the incomes of the three?

(a) ₹ 4800 (b) ₹ 5600 (c) ₹ 4200 (d) ₹ 5000

SOLUTIONS

1. (d) : $(2 + 1) = 3 = 2^2 - 1$
$(2 + 1)(2^2 + 1) = 3 \times 5 = 15 = 2^4 - 1$
$(2 + 1)(2^2 + 1)(2^4 + 1) = 3 \times 5 \times 17 = 2^8 - 1$
So, $(2 + 1)(2^2 + 1)(2^4 + 1) \ldots\ldots (2^{128} + 1) = 2^{256} - 1$

2. (a) : Let x be the total number of people the college will ask for donations.
$\because$ People already solicited = $0.6x$
Amount raised from the people solicited
$= 600 \times 0.6x = 360x$
Now, $360x$ constitutes 75% of the amount.
Hence, remaining 25% = $120x$
$\therefore$ Average donation from remaining people

$$= \frac{120x}{0.4x} = ₹300$$

3. (b) : Scheme A : $R = 12\%$, $t = 2$ year, $I = 3600$

$$3600 = \frac{A \times 12 \times 2}{100}$$

$A = 15,000$

Amount invested in $B = 35000 - 15000 =$ ₹ 20000

Scheme B : $R = 10\%$, $P = 20,000$

$\text{CI} = (1.1)^2 \times 20000 - 20000 = 20000\,(1.21 - 1)$

$$= 20000 \times \frac{21}{100} = 4200$$

CI = 4,200

So, Interest accrued on scheme B is ₹ 4200.

4. **(c) :** If A is the sum of all the alternate digits of the number starting from the units place and B is the sum of all the alternate digits of the number starting from tens place, then remainder by 11 is given by the number $A - B$.

For one group of ten digits of the number, $A - B = -5$

For the 50 groups, it is -250.

So, when this number is divided by 11, the remainder will be -8 or $+3$.

Therefore 8 has to be added to the number, so that sum is divisible by 11.

5. **(d) :** Let manufacturer C.P = ₹ 100

Then, his S.P would be $= 1.1 \times 100 =$ ₹ 110. It is distributor's C.P.

Then, distributor S.P $= 1.1 \times 110 =$ ₹ 121.

Dealer S.P $= 1.2 \times 121 =$ ₹ 145.2.

Retailer S.P $= 1.2 \times 145.2 =$ ₹ 174.24.

After discount $= 0.9 \times 174.24 =$ ₹ 156.816.

If he had bought from distributor he would save $= 35.816$

$$\text{Percentage reduction} = \frac{35.816}{156.816} \times 100 = 22.8\%$$

6. **(a) :** Amount reinvested in Debit and Equity is in the ratio 6 : 7.

Let the amount invested in Equity be $7x$ and amount invested in Debit will be $6x$.

Now, $7x =$ ₹ $94500 \Rightarrow x = 13500$

Amount reinvested in Debit $= 6 \times 13500 =$ ₹ 81000

Total amount reinvested $= 94500 + 81000 =$ ₹ 175500.

As this is the original amount + 30% dividend on the original amount.

Let original amount be ₹ y

$$\Rightarrow y + \frac{30}{100} \times y = ₹\ 175500 \Rightarrow 1.3y = 175500$$

$\Rightarrow y = 135000$

Hence, total amount invested is ₹ 135000.

$\therefore$ Amount invested in Equity is $\frac{5}{9} \times 135000 =$ ₹ 75000

Hence amount invested in Equity is ₹ 75000.

7. **(c) :** The marks of the 5 student and the remainder that they leave when divided by 3 are tabulated below.

Marks	71	76	80	82	91
Remainder	2	1	2	1	1

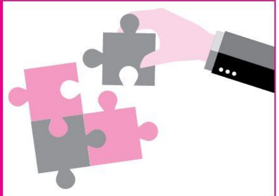
After the first 3 marks are entered, the total has to be a multiple of 3. The remainders can only be 1, 1, 1. (2, 1, 1 or 2, 2, 1 would not produce a multiple of 3).

So, the fourth and fifth marks to be entered were 71 and 80.

8. **(c) :**

Wine	**Water**	
6 L	18 L	
1 :	3	
25%	75%	(Original ratio)
50%	50%	(Required ratio)

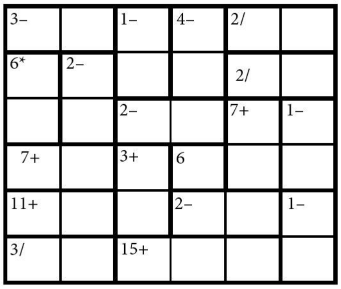
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3–		1–	4–	2/	
6*	2–			2/	
		2–		7+	1–
7+		3+	6		
11+			2–		1–
3/		15+			

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

In this case, the percentage of water is being reduced when the mixture is being replaced with wine.

So, the ratio of left quantity to the initial quantity is 2 : 3.

Therefore, $\frac{2}{3} = \left[1 - \frac{k}{24}\right] \Rightarrow \frac{k}{24} = \frac{1}{3} \Rightarrow k = 8$ litres

9. (b) : $15\,m = 24\,w = 36\,b$ ...(1)

Total work = 15 × 12 × 8 = 1440 men hours

New work = $\frac{9}{4} \times 1440 = 3240$ men hours.

Now, x men + 12 women + 6 boys

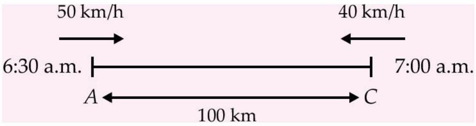
$= x$ men + $\frac{15}{2}$ men + $\frac{15}{6}$ men [Using (1)]

$= \frac{6x+60}{6}$ men are working on 3240 units.

Number of men = $\frac{3240}{6 \times 30} \Rightarrow \frac{6x+60}{6} = 18$

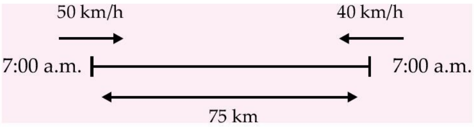
$\Rightarrow 6x = 108 - 60 \Rightarrow x = 8$ men

10. (a) :



From 6:30 to 7:00 *i.e.* for half hour, the train from Ahmedabad will move for half hour and will cover a distance of $50 \times \frac{1}{2}$ *i.e.* 25 km.

Now, past 7:00 a.m. both the trains will move.



Now, both the trains are moving in the opposite direction. So, they will collide after

$= \frac{75}{50 + 40} = \frac{75}{90} = \frac{5}{6}$ hr or 50 minutes

So, they will collide 50 minutes after 7:00 a.m. *i.e.* at 7:50 a.m.

Mr. Shah came to know at 7:30 that both the trains are on the same track, so he has 20 minutes to avoid the collision between the two trains.

11. (d) : The number of terms of the series forms the sum of first n natural numbers *i.e.* $\frac{n(n+1)}{2}$.

Thus the first 23 letters will account for the first $= \frac{23 \times 24}{2} = 276$ terms of the series.

The 288th term will be the 24th letter which is x.

12. (b) : The S.P. of DC = ₹ 18 per kg

The S.P. of VC = ₹ 30 per kg

The C.P. of DC = ₹ 20 per kg

The C.P. of VC = ₹ 25 per kg

The S.P. of mixture = ₹ 27.5 per kg

The C.P. of mixture = ₹ 22 per kg

$$22 = 20x + 25(1-x) \Rightarrow x = \frac{3}{5} \Rightarrow 1 - x = \frac{2}{5}$$

Therefore, the ratio of DC is to VC is 3 : 2.

13. (d) : Assume that the radii of each of the spheres and the can are 1, then the volume of each ball is $\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$ and the total volume of 3 balls is $3\left(\frac{4}{3}\pi\right) = 4\pi$. Since the height of the can is 6 (the diameter of each sphere is 2), the volume of the can is $\pi(1)^2\,(6) = 6\pi$

So, the balls take up $\frac{4\pi}{6\pi} = \frac{2}{3}$ of the can.

14. (d) : The given word SERIES, this word contains 2S, 2E and rest are distinct.

Case 1: 4 Letters are distinct

Using S, E, R, I

Number of ways = 4! = 24

Case 2 : 2 letters are same and rest are distinct

Using SS or EE and using two letter out of remaining R and I.

Ways = ${}^2C_1 \times {}^3C_2 \times \frac{4!}{2!}$ = 72 ways

Case 3 : SS and EE

Number of ways = $\frac{4!}{2!2!}$ = 6 ways

Total number of ways = 24 + 72 + 6 = 102

15. (c) : Total players = 15

Total number of selected players = 11

∴ Total possible outcomes = ${}^{15}C_{11} = 1365$

Favourable number of cases

= n(3 bowlers) + n(4 bowlers) + E(5 bowlers)

(∵ Total number of bowlers = 5)

$= ({}^5C_3 \times {}^{10}C_8) + ({}^5C_4 \times {}^{10}C_7) + ({}^5C_5 \times {}^{10}C_6)$

$= (10 \times 45) + (5 \times 120) + (1 \times 210) = 1260$

∴ $P(\text{at least 3 bowlers}) = \frac{1260}{1365} = \frac{12}{13}$

16. (b) : A = 8 glasses in 48 minutes
B = 16 glasses in 2 hours
C = 4 glasses in 40 minutes

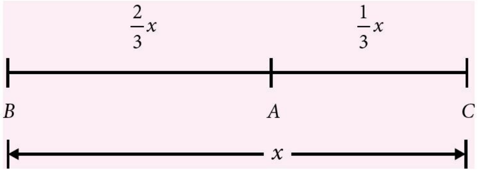
In 4 hours, $A = \frac{4\times60}{48}\times8 = 40$ glasses

$B = \frac{4\times60}{120}\times16 = 32$ glasses

$C = \frac{4\times60}{40}\times4 = 24$ glasses

Total water = 40 + 32 + 24 = 96 glasses.
Capacity = 96 × 0.5 L = 48 litres

17. (d) :



Let distance between B and C be x, $AB = \frac{2}{3}x$

$\therefore \quad AC = \frac{1}{3}x$

Speed of boat upstream = 16 – 2 = 14 km/h
Speed of boat downstream = 16 + 2 = 18 km/h

$$\frac{2x}{3\times14}+\frac{x}{18}=\frac{13}{2};\ \frac{1}{2}\left[\frac{2x}{3\times7}+\frac{x}{9}\right]=\frac{13}{2};\ \frac{18x+21x}{3\times7\times9}=13$$

$39x = 3\times7\times9\times13$; $x = 63$ km

$\therefore$ Total distance travelled

$$=\frac{2}{3}x+x=\frac{5}{3}x=\frac{5}{3}\times63 = 105 \text{ km.}$$

18. (a) : Let the price of 1 g be ₹ 1.
The shopkeeper buys 1100 g at the rate of ₹ 1000.
He sells 900 g at the cost price of ₹ 1000.

So, CP for 900 g would be $=\frac{1000}{1100}\times900$ = ₹ 818.18

So, gain % $=\frac{1000-818.18}{818.18}\times100=22.22\%=\frac{200}{9}\%$

19. (b) : Compound interest received after 2 year = ₹ 1408
Let the principal invested in scheme B is x at rate = 20%
Now, $1408 = x[(1.2)^2 - 1]$

$1408 = x\,[1.44 - 1];\ x=\frac{1408}{0.44}=3200$

Amount invested in scheme B is ₹ 3200.

Now, interest received in scheme $A = 1500\times\frac{5}{100}\times14$
= 70 × 15 = ₹ 1050

Amount received from scheme A = 1500 + 1050 = ₹ 2550
Now, additional amount that is added is
3200 – 2550 = ₹ 650

20. (a) : Let a, b, and c be the annual incomes of Ram, Shyam, and Pratap, respectively.
Now, we have given that
The arithmetic mean of the annual incomes of Ram and Shyam was ₹ 3800. Hence, $\left(\frac{a+b}{2}\right)=3800$

Multiplying by 2 yields $a + b = 2\times3800 = 7600$.
The arithmetic mean of the annual incomes of Shyam and Pratap was ₹ 4800.

Hence, $\left(\frac{b+c}{2}\right) = 4800$

Multiplying by 2 yields $b + c = 2 \times 4800 = 9600$.
The arithmetic mean of the annual incomes of Pratap and Ram was ₹ 5800.

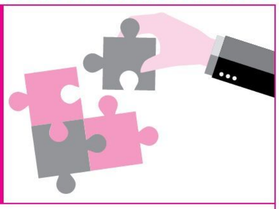
Hence, $\left(\frac{c+a}{2}\right) = 5800$

Multiplying by 2 yields $c + a = 2 \times 5800 = 11{,}600$
Summing these three equations yields:
$(a + b) + (b + c) + (c + a) = 7600 + 9600 + 11600$
$2a + 2b + 2c = 28{,}800$; $a + b + c = 14{,}400$
The average of the incomes of the three

$=\left(\frac{a+b+c}{3}\right)=\frac{14400}{3}$ = ₹ 4800

◆ ◆

PUZZLE CORNER



ANSWER · FEBRUARY 2023

2÷ 2	1	3÷ 6	2÷ 3	8+ 4	2− 5
1− 5	4	2	6	1	3
3− 4	6 6	40× 5	2	3	9+ 1
1	2− 5	3	4	2	6
54× 6	3	3− 4	1	3− 5	2
3	1− 2	1	15+ 5	6	4

Brain Teaser

LOGICAL REASONING

for Various Competitive Exams

Direction Q.(1 and 2) : What will come in place of '?' in the given series?

1. 9, 31, 73, 141,?
(a) 164 (b) 280 (c) 239 (d) 241

2. A2E, C5H, E12K, G27N,?
(a) I58Q (b) I77R (c) J58R (d) J57Q

Direction Q.(3 and 4) : The two words or numbers on the right of (: :) are related in the some way as the words on the left of (: :). Identify what will come in place of blank.

3. GRIN : RING :: ……………: NECKLACE
(a) NEECKLAC (b) LACENECK
(c) ENECKLAC (d) NECKLAEC

4. 216 : 81 :: 512 : ……………
(a) 100 (b) 64 (c) 48 (d) 256

5. In a code language, 'FUN' is coded as 'HWP', then 'TCFKQ' is code of
(a) REALER (b) RIDER
(c) RADIO (d) ROMAN

6. If LUXOR is coded as 30, then GUILDS will be coded as
(a) 40 (b) 36 (c) 38 (d) 24

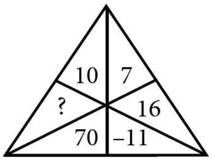
Direction Q.(7 and 8) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?

7.
(a) Bagel (b) Bun (c) Partly (d) Damper

8.
(a) Cuckoo (b) Crow (c) Bat (d) Parrot

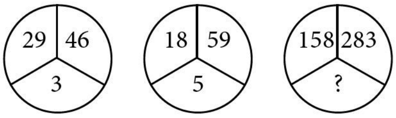
Direction Q.(9 and 10): In each of the following figures, find the number which replace the sign of '?'

9.



(a) −173 (b) 173 (c) 713 (d) 143

10.



(a) 5 (b) 2 (c) 7 (d) 9

Direction Q.(11 and 12) : Study the following information carefully and answer the questions given below:

P is son of Q. Q is mother of R. R is wife of T. T is father of V. V is brother of W. Y is mother of T.

11. Which of the following is true based upon the relationships given above?
(a) W is granddaughter of Y
(b) R is sister of P
(c) V is son of Q
(d) V is brother-in-law of Y

12. Which of the following is/are required to establish that W is the daughter of R?
(a) R has only three children, one son and two daughters
(b) Q has only one grandson
(c) Y has only two children, a son and a daughter
(d) Either (a) or (b)

13. Bimla left her house to go to a temple. She walked 10 m straight to north and then took a left to walk 3 m and then a right to walk 5 m and then a left to walk 2 m and then a right to walk 5 m and then again a right to walk 30 m to reach the temple. How far is the temple and in which direction from Bimla's house?
(a) 25 m North (b) $15\sqrt{2}$ m North- East
(c) $5\sqrt{41}$ m North- East (d) $15\sqrt{2}$ m North

Direction Q.(14 and 15) : In these questions, relationship between different elements is shown in the statements. The statements are followed by two conclusions.

Mark answer
(a) if only conclusion (I) is true.
(b) if only conclusion (II) is true.
(c) if neither conclusion (I) nor (II) is true.
(d) if both conclusions (I) and (II) are true.

14. Statements : $G \le K = O \ge U > P < S$

Conclusions :

I. $G \le U$ II. $S > O$

15. Statements: $O = L \le T > S$; $L > I$; $T \le Z$

Conclusions :

I. $Z > I$ II. $S < I$

16. Vinay remembers that his brother Anupam's birthday is after 20th of August but before 28th of August, while Rita remembers that Anupam's birthday is before 22nd of August but after 12th of August. On which date does Anupam's birthday fall?

(a) 20th August (b) 21th August

(c) 22nd August (d) Cannot be determined

Direction Q.(17 and 19) : Study the following information carefully and answer the questions given below:

Each of the six friends, A, B, C, D, E and F scored different marks in an examination. C scored more than only A and E. D scored less than only B. E did not score the least. The one who scored the third highest marks scored 81 marks. E scored 62 marks.

17. Which of the following could possibly be C's score?

(a) 70 (b) 94 (c) 86 (d) 61

18. Which of the following is true with respect to the given information?

(a) D's score was definitely less than 60

(b) F scored the maximum marks

(c) Only two people scored more than C

(d) There is a possibility that B scored 89 marks

19. The person who scored the maximum, scored 13 marks more than F's marks. Which of the following can be D's score?

(a) 94 (b) 60 (c) 88 (d) 78

20. Seventeen people are standing in a straight line facing south. What is Bhavna's position from the left end of the line?

I. Sandeep is standing second to the left of Sheetal. Only five people stand between Sheetal and the one who is standing at the extreme right end of the line. Four people stand between Sandeep and Bhavna.

II. Anita is standing fourth to the left of Sheetal. Less than three people are standing between Bhavna and Anita.

(a) if the data in statement (I) alone are sufficient to answer the question, while the data in statement (II) alone are not sufficient to answer the question.

(b) if the data in statement (II) alone are sufficient to answer the question, while the data in statement (I) alone are not sufficient to answer the question.

(c) if the data either in statement (I) alone or in statement (II) alone are sufficient to answer the question.

(d) if the data given in both the statements (I) and (II) together are not sufficient to answer the question.

SOLUTIONS

1. (d) : $(2)^3 + (1)^2 \to (3)^3 + (2)^2 \to (4)^3 + (3)^2 \to (5)^3 + (4)^2 \to (6)^3 + (5)^2$

$$9 \longrightarrow 31 \longrightarrow 73 \longrightarrow 141 \longrightarrow \boxed{241}$$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

9				7				2				9				5				7
	3						9						5						9	
		4				6								4				6		
			7	3	1										7	1	4			
4			9		6			8				1			3		6			5
			4	2	8										5	9	8			
		1				5								3				1		
	4						3						4						3	
5				9				7				8				4				2
									9	3	2									
									7		8									
									1	6	5									
9				4				5				9				5				1
	5						2						1						6	
		1				3								5				2		
			5	7	2										1	7	9			
3			6		9			4				3			6		8			9
			4	1	3										5	2	3			
		8				7								8				1		
	9						1						6						4	
4				6				2				4				9				5

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

2. (a) :

$$\begin{tikzcd}
A \arrow[d, "+2"] & 2 \arrow[d, "\times 2+1"] & E \arrow[d, "+3"] \\
C \arrow[d, "+2"] & 5 \arrow[d, "\times 2+2"] & H \arrow[d, "+3"] \\
E \arrow[d, "+2"] & 12 \arrow[d, "\times 2+3"] & K \arrow[d, "+3"] \\
G \arrow[d, "+2"] & 27 \arrow[d, "\times 2+4"] & N \arrow[d, "+3"] \\
\boxed{I} & \boxed{58} & \boxed{Q}
\end{tikzcd}$$

3. (c) : The relationship between GRIN and RING is same as 4123 and 1234. Last letter becomes the first letter and rest remain as it is.

4. (d) : The relationship is $x^3 : (x/2)^4$.

5. (c) : $F \xrightarrow{+2} H$, $U \xrightarrow{+2} W$, $N \xrightarrow{+2} P$

Similarly, $T \xrightarrow{-2} R$, $C \xrightarrow{-2} A$, $F \xrightarrow{-2} D$

$K \xrightarrow{-2} I$, $Q \xrightarrow{-2} O$

6. (d) : As,

$$\frac{L+U+X+O+R}{3} = \frac{12+21+24+15+18}{3} = \frac{90}{3} = 30$$

In the same way, $\frac{G+U+I+L+D+S}{3}$

$$= \frac{7+21+9+12+4+19}{3} = \frac{72}{3} = 24$$

7. (c) : Partly is not a type of bread.

8. (c) : Except Bat, all others are birds. Bat is a mammal.

9. (a) : $10 + (-3)^1 = 7$; $7 + (-3)^2 = 7 + 9 = 16$

$16 + (-3)^3 = 16 - 27 = -11$

$-11 + (-3)^4 = -11 + 81 = 70$

$70 + (-3)^5 = 70 - 243 = -173$

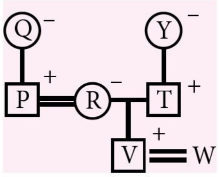
10. (d) : $29 + 46 = 75$

Now, $7 + 5 = 12$ and $1 + 2 = 3$; $18 + 59 = 77$

Now, $7 + 7 = 14$ and $1 + 4 = 5$

So, $158 + 283 = 441$; $4 + 4 + 1 = 9$

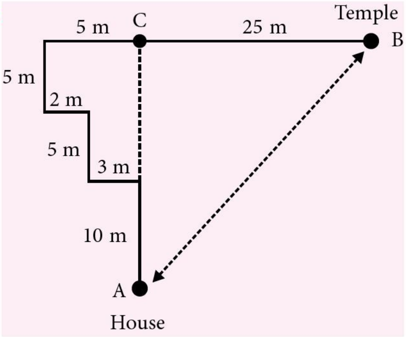
(11-12) :



11. (b) : Clearly, R is sister of P.

12. (d): Either (a) or (b)

13. (c) :



Thus, $AB^2 = AC^2 + BC^2 = (20)^2 + (25)^2$

$\Rightarrow AB^2 = 1025 \Rightarrow AB = 5\sqrt{41}$ m

According to the figure, direction is North-East.

14. (c) : Statements: $G \le K = O \ge U > P < S$

Conclusions:

I. $G \le U$: Not true II. $S > O$: Not true

Hence, neither conclusion (I) nor (II) is true.

15. (a) : Conclusions:

I. $Z > I$: True II. $S < I$: Not true

Hence, only conclusion (I) is true.

16. (b) : According to Vinay, his brother Anupam's birthday may be on 21st, 22nd, 23rd, 24th, 25th, 26th, or 27th August.

According to Rita, Anupam's birthday may be on 13th, 14th, 15th, 16th, 17th, 18th, 19th, 20th or 21st August.

Hence, the common date is 21st August.

(17 - 19) : $B, D, F > C > A, E$

$B > D > F > C > A, E$; $B > D > F > C > E > A$

Score of F = 81; Score of E = 62

17. (a) : C scored more than 62 and less than 81.

$62 < 70 < 81$

18. (d) : D's score was more than 81 but less them B score. B scored the maximum marks.

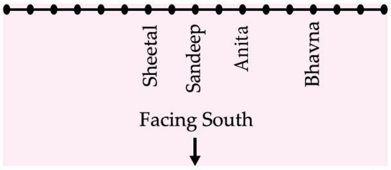
If B scored 89, then there is a possibility that D's score can be 88, 87, 86, 85, 84, 83 and 82.

19. (c) : Score of F = 81; Score of B = 81 + 13 = 94

Therefore, score of D = more than 81 and less than 94

$81 < 88 < 94$

20. (a) : From statement (I),



We can conclude that Bhavna's position is 4th from the left end. From statement (II), nothing can be concluded.

◆

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. The domain of $f(x)=\sqrt{\left(\frac{1}{\sqrt{x}}-\sqrt{(x+1)}\right)}$ is

(a) $x > -1$ (b) $(-1, \infty)\backslash\{0\}$

(c) $\left(0, \frac{\sqrt{5}-1}{2}\right]$ (d) $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

2. The solution of system of equations $x + (\sin\alpha)y = 1$ and $(\sin\alpha)x + 4y = 2$ satisfy $x \ge \frac{4}{5}$ & $y \le \frac{1}{2}$, then

(a) $\alpha \in \left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ (b) $\alpha \in \left[0, \frac{\pi}{6}\right]$

(c) $\alpha \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) None of these

3. If $f(x)=\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}, x \ne a,$ the value which should be assigned to f at $x = a$ so that it is continuous every where is

(a) $2/\pi$ (b) $e^{-2/\pi}$ (c) 2 (d) $e^{2/\pi}$

4. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}$, $\vec{b} = \vec{\alpha} + 3\vec{\beta}$, If $|\vec{\alpha}| = |\vec{\beta}| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\pi/3$, then length of a diagonal of the parallelogram is

(a) $4\sqrt{5}$ units (b) $3\sqrt{2}$ units

(c) $4\sqrt{3}$ units (d) $5\sqrt{3}$ units

5. If 1, α_1, α_2, ..., α_{2008} are (2009)th roots of unity, then the value of $\sum_{r=1}^{2008} r(\alpha_r + \alpha_{2009-r})$ equals

(a) 2009 (b) 2008 (c) 0 (d) –2009

6. In the next world cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, when each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next world cup will be

(a) 54 (b) 53 (c) 52 (d) 51

7. If X denote the number of times heads occur in n tosses of a fair coin. If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in A.P.; then value of n is

(a) 7, 14 (b) 10

(c) 12 (d) none of these

8. The angle between the pair of tangents drawn from the point (2, 4) to the circle $x^2 + y^2 = 4$ is

(a) $\tan^{-1}(3/8)$ (b) $\tan^{-1}(4/3)$

(c) 90° (d) None of these

9. If $x \in (-\infty, -2)$ and $y^3 - 3y + x = 0$, then

(a) y is not a monotonic function

(b) y is an increasing function of x

(c) y is not a function of x

(d) y is a decreasing function of x

10. The value of the integral $\int \frac{\sin^3 x + \sin^5 x}{\cos^2 x + \cos^4 x} dx$ is

(a) $-\cos x + 2(\cos x)^{-1} + 6\tan^{-1}(\cos x) + C$

(b) $\cos x - 2(\cos x)^{-1} + 4\tan^{-1}(\cos x) + C$

(c) $\cos x - 2(\cos x)^{-1} + \tan^{-1}(\cos x) + C$

(d) $\sec x + \tan x \sin x - 6\tan^{-1}(\cos x) + C$

SECTION-II

Numerical Answer Type

11. If $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \ne \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to ________.

12. If the greatest and least values of the function $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 2]$ are λ and μ, then the value of $\lambda^4 + \mu^8$ must be ______.

13. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., (where a and b are two n on zero red numbers) then $a^{-1} - b^{-1}$ is equal to ______.

14. If $f : R \to R$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in R$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^{3} f(x)dx$, then the value of $I_1 + 2I_2$ is equal to _______.

15. A tangent is drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x and y axes at A and B respectively such that $AP : PB = 1 : 3$. If $f(1) = 1$, then the curve passes through $\left(k, \frac{1}{8}\right)$, where $k =$ ______ .

SOLUTIONS

1. (c) : Here $f(x) = \sqrt{\left(\frac{1}{\sqrt{x}} - \sqrt{(x+1)}\right)}$

$f(x)$ will be defined, if

$$\frac{1}{\sqrt{x}} - \sqrt{x+1} \ge 0,\ x \ne 0,\ x + 1 \ge 0 \text{ and } x > 0$$

$$\Rightarrow 1 - \sqrt{x^2 + x} \ge 0 \Rightarrow \sqrt{x^2 + x} \le 1 \Rightarrow x^2 + x - 1 \le 0$$

$$\Rightarrow \left(x + \frac{1-\sqrt{5}}{2}\right)\left(x + \frac{1+\sqrt{5}}{2}\right) \le 0$$

$$\Rightarrow \frac{-1-\sqrt{5}}{2} \le x \le \frac{\sqrt{5}-1}{2} \qquad \text{...(i)}$$

Also, $x + 1 \ge 0 \Rightarrow x \ge -1$ and $x > 0$ ...(ii)

From (i) and (ii), $x \in \left(0, \frac{\sqrt{5}-1}{2}\right)$

2. (b)

3. (d) : For $f(x)$ to be continuous, we must have

$$f(a) = \lim_{x\to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$

Now, $\log f(x) = \tan\left(\frac{\pi x}{2a}\right)\log\left(2 - \frac{x}{a}\right)$

$$\Rightarrow \lim_{x\to a}\log f(x) = \frac{\log\left(2 - \frac{x}{a}\right)}{\cot\left(\frac{\pi x}{2a}\right)} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to a}\frac{\frac{-1/a}{2 - x/a}}{-\frac{\pi}{2a}\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right)} \qquad \text{(Using } L' \text{ Hospital's Rule)}$$

$$= \frac{2}{\pi}\lim_{x\to a}\frac{1}{2 - \frac{x}{a}} \times \lim_{x\to a}\sin^2\left(\frac{\pi x}{2a}\right) = \frac{2}{\pi}$$

Hence, $\lim_{x\to a} f(x) = e^{2/\pi} \Rightarrow f(a) = e^{2/\pi}$

4. (c) : Let $AB = \vec{a} = 3\vec{\alpha} - \vec{\beta},\ BC = \vec{b} = \vec{\alpha} + 3\vec{\beta}$

Diagonal, $AC = AB + BC = \vec{a} + \vec{b}$

$$\Rightarrow |AC| = |\vec{a} + \vec{b}| \Rightarrow |AC| = |4\vec{\alpha} + 2\vec{\beta}|$$

$$\Rightarrow |AC|^2 = 16|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 + 16\vec{\alpha}\cdot\vec{\beta}$$

$$\Rightarrow |AC|^2 = 64 + 16 + 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$$

$$= 80 + 16 \times 4 \times \frac{1}{2} = 112 \Rightarrow |AC| = 4\sqrt{7} \text{ units}$$

Other diagonal is $|BD| = |\vec{a} - \vec{b}|$

$$\Rightarrow |BD|^2 = |2\vec{\alpha} - 4\vec{\beta}|^2 = 4|\vec{\alpha}|^2 + 16|\vec{\beta}|^2 - 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$$

$$\Rightarrow |BD|^2 = 16 + 64 - 64\left(\frac{1}{2}\right) \Rightarrow |BD| = \sqrt{48} = 4\sqrt{3} \text{ units}$$

5. (d)

6. (b) : The number of matches in first round $= {}^6C_2 + {}^6C_2 = 30$

The number of matches in next round $= {}^6C_2 = 15$ and the number of matches in the semifinal round $= {}^4C_2 = 6$

Number of matches in the final round = 2

Hence, required number of matches = 30 + 15 + 6 + 2 = 53.

7. (a) : Clearly, X is a binomial variate with parameters n and $p = 1/2$ such that $P(X = r)$

$$= {}^nC_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r} = {}^nC_r\left(\frac{1}{2}\right)^n$$

Now, $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in A.P.

$$\Rightarrow 2P(X = 5) = P(X = 4) + P(X = 6)$$

$$\Rightarrow 2\cdot{}^nC_5\left(\frac{1}{2}\right)^n = {}^nC_4\left(\frac{1}{2}\right)^n + {}^nC_6\left(\frac{1}{2}\right)^n$$

$$\Rightarrow 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow 2\frac{n!}{(n-5)!\,5!} = \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!}$$

$$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{6\times 5}$$

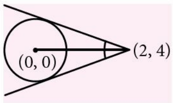
$\Rightarrow\ n^2 - 21n + 98 = 0 \Rightarrow (n-7)(n-14) = 0$

$\Rightarrow\ n = 7$ or 14.

8. (b) : Equation of pair of tangents to a circle is $T^2 = SS_1$, where $T = xx_1 + yy_1 - 4$

$S_1 = x_1^2 + y_1^2 - 4$

$S = x^2 + y^2 - 4$



$\therefore\ (2x + 4y - 4)^2 = (x^2 + y^2 - 4)(4 + 16 - 4)$

$\Rightarrow\ 4(x + 2y - 2)^2 = 16(x^2 + y^2 - 4)$

$\Rightarrow\ x^2 + 4y^2 + 4xy - 8y - 4x + 4 = 4x^2 + 4y^2 - 16$

$\Rightarrow\ 3x^2 - 4xy + 8y + 4x - 20 = 0$

$= ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$$\therefore\ \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\times 2}{3} \Rightarrow \theta = \tan^{-1}(4/3)$$

9. (d) : $y^3 - 3y + x = 0$

$\Rightarrow\ x = -y^3 + 3y < -2 \quad (\because x \in (-\infty, -2))$

$\Rightarrow\ y^3 - 3y - 2 > 0 \Rightarrow (y+1)^2(y-2) > 0 \therefore y > 2$

Now, $x = -y^3 + 3y$

$$\therefore\ \frac{dx}{dy} = -3y^2 + 3 = 3(1 - y^2)$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{1}{3(1-y^2)} < 0 \quad (\text{as } y > 2)$$

$\Rightarrow$ y is a decreasing function of x.

10. (a)

11. (144) : Let $A(r\cos\alpha, r\sin\alpha)$, $B(r\cos\beta, r\sin\beta)$ and $C(r\cos\gamma, r\sin\gamma)$.

Since, circumcentre and orthocentre both lies on y-axis.

$\therefore$ Centroid also lies on y-axis.

$\Rightarrow\ \cos\alpha + \cos\beta + \cos\gamma = 0$

$\therefore\ \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha \cdot \cos\beta \cdot \cos\gamma$ ...(i)

Now, $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma)$

$$\Rightarrow\ \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos^3\alpha + \cos^3\beta + \cos^3\gamma} = 4 \quad \text{...(ii)}$$

Using (i) and (ii), we get

$$\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cdot \cos\beta \cdot \cos\gamma}\right)^2 = 12^2 = 144$$

12. (626) : Given, $f(x) = x^3 - 6x^2 + 9x + 1$

$\therefore\ f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

For maxima or minima put $f'(x) = 0$

$\Rightarrow\ x = 1, 3$ but $3 \notin [0, 2]$

$\therefore$ Only one critical point in $[0, 2]$ is $x = 1$.

Now, greatest value, $\lambda = \max.\ \{f(0), f(1), f(2)\} = \max.\ \{1, 5, 3\} = 5$

and least value, $\mu = \min.\ \{f(0), f(1), f(2)\} = \min.\ \{1, 5, 3\} = 1$

$\therefore\ \lambda^4 + \mu^8 = 5^4 + 1^8 = 625 + 1 = 626$

13. (38) : p and r are the roots of the equation, $x^2 - 8ax + 2a = 0$

$\therefore\ p + r = 8a,\ pr = 2a \Rightarrow 1/p + 1/r = 4$ ...(i)

Also, q and s are the roots of $x^2 + 12bx + 6b = 0$

$\therefore\ q + s = -12b,\ qs = 6b \Rightarrow 1/q + 1/s = -2$ ...(ii)

Given that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P.

$$\therefore\ \frac{1}{q} - \frac{1}{p} = \frac{1}{r} - \frac{1}{q} \Rightarrow \frac{2}{q} = \frac{1}{r} + \frac{1}{p} = 4 \quad \text{[Using (i)]}$$

$$\Rightarrow\ q = \frac{1}{2} \text{ and } s = \frac{-1}{4}$$

$$\text{Clearly, } \frac{1}{r} - \frac{1}{q} = \frac{1}{s} - \frac{1}{r} \Rightarrow \frac{2}{r} = \frac{1}{q} + \frac{1}{s}$$

$$\Rightarrow\ \frac{2}{r} = -2 \Rightarrow r = -1 \text{ and } p = \frac{1}{5} \quad \text{[Using (ii)]}$$

$$\therefore\ \frac{1}{a} - \frac{1}{b} = \frac{2}{pr} - \frac{6}{qs} = 38$$

14. (16) : We have, $f(x) + f(x+1) = 2$ ...(i)

Replacing x with $x + 1$, we get

$f(x+1) + f(x+2) = 2$ ...(ii)

From (i) and (ii), we have $f(x) = f(x+2)$

$\Rightarrow$ $f(x)$ is periodic with period 2.

$$\text{Now, } I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$$

$$\text{and } I_2 = \int_{-1}^3 f(x)dx = 2\int_0^2 f(x)\,dx \Rightarrow I_1 = 2I_2$$

$$\therefore\ I_1 = 4\left[\int_0^1 f(x)dx + \int_1^2 f(x)dx\right]$$

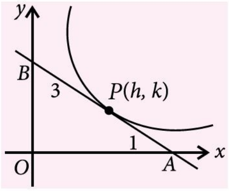
$$= 4\left[\int_0^1 f(x)dx + \int_0^1 f(x+1)dx\right]$$

$$= 4\left[\int_0^1 f(x)dx + \int_0^1 (2 - f(x))dx\right] \quad \text{[Using (i)]}$$

$$= 4\left[\int_0^1 f(x)dx + \int_0^1 2dx - \int_0^1 f(x)dx\right] = 8$$

$\Rightarrow\ I_1 = 8 \Rightarrow I_2 = 4 \ \therefore\ I_1 + 2I_2 = 8 + 8 = 16$

15. (2) : $\frac{3x}{h} + \frac{y}{k} = 1$ is the equation of the tangent whose slope is $-3k/h$.



$$\frac{dy}{dx} = -\frac{3y}{x} \Rightarrow x\frac{dy}{dx} + 3y = 0$$

$$\Rightarrow\ \frac{dy}{y} = -3\frac{dx}{x} \Rightarrow \log y = -3\log x + \log c$$

$\Rightarrow\ y = cx^{-3} \Rightarrow y = \frac{c}{x^3}$, which passes through (1,1)

$\Rightarrow\ c = 1$

$\therefore$ Equation of the curve is $y = 1/x^3$.

$$\because\ \text{It passes through } \left(k, \frac{1}{8}\right) \therefore\ \frac{1}{8} = \frac{1}{k^3} \Rightarrow k = 2$$

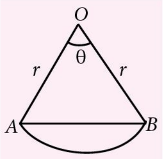
YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If a circular sector of perimeter 60 meter with maximum area is to be constructed, then what will be the radius of the circular arc in meter. ***(Anushka, Karnataka)***

Ans. Let OAB be the circular sector.



Perimeter of OAB = 60 m

$$\therefore \quad \frac{\theta}{360}\times 2\pi r + 2r = 60$$

$$\Rightarrow \quad \frac{\theta}{360} = \frac{60-2r}{2\pi r} \qquad \text{...(i)}$$

We need maximum area, and we know that area of a sector $(A) = \dfrac{\theta}{360}\times \pi r^2$

$$i.e., \quad A = \frac{60-2r}{2\pi r}\times \pi r^2 \qquad \text{(Using (i))}$$

$$\Rightarrow \quad A = 30r - r^2$$

Differentiating w.r.t 'r', $\dfrac{dA}{dr} = 30 - 2r$

For critical point, we have $\dfrac{dA}{dr} = 0 \Rightarrow 30 - 2r = 0$

$$\Rightarrow \quad r = \frac{30}{2} = 15\text{ m}. \text{ Now, } \frac{d^2A}{dr^2} = -2 < 0$$

$\therefore \quad r = 15$ m gives the maximum area.

2. If $\vec{a} = -\hat{i} - \hat{k}, \hat{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors and $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\times\vec{a} = 0$, then show that $\vec{r}\cdot\vec{b} = 9$. ***(Rakesh, Assam)***

Ans. We have, $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}, \vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Now given that, $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$

So, $\vec{c}\times\vec{b} = -3\hat{i} - 3\hat{j} + 3\hat{k}$ ...(i)

and $\vec{r}\times\vec{b} = -z\hat{i} - z\hat{j} + (x+y)\hat{k}$ ...(ii)

On comparing (i) and (ii), we get $z = 3$

and $x + y = 3$ ...(iii)

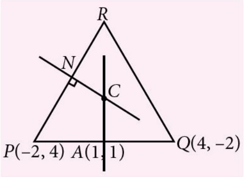
Now, $\vec{r}\cdot\vec{a} = 0 \Rightarrow -x - z = 0 \Rightarrow x = -z \Rightarrow x = -3$

From (iii), $y = 3 - x = 3 - (-3) = 6$

$\Rightarrow \quad \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

Now, $\vec{r}\cdot\vec{b} = (-3)(-1) + 6\cdot 1 = 3 + 6 = 9$

3. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively and equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then find the centre of the circumcircle of the ΔPQR. ***(Vibhor Singh, A.P.)***



Ans. Let A is mid point of PQ.

So, coordinates of point A

$$= \left(\frac{-2+4}{2}, \frac{4-2}{2}\right) \; i.e., \; (1, 1)$$

$$\text{Slope of } PQ = \frac{4+2}{-2-4} = -1$$

$\Rightarrow$ Slope of perpendicular bisector of $PQ = 1$

So, equation of perpendicular bisector of PQ is

$(y - 1) = 1(x - 1) \Rightarrow x - y = 0$ ...(i)

Also, equation of perpendicular bisector of PR is

$2x - y + 2 = 0$ ...(ii)

Solving (i) and (ii), we get $x = -2$ and $y = -2$

$\Rightarrow$ Centre of circumcircle of ΔPQR is (–2, –2).

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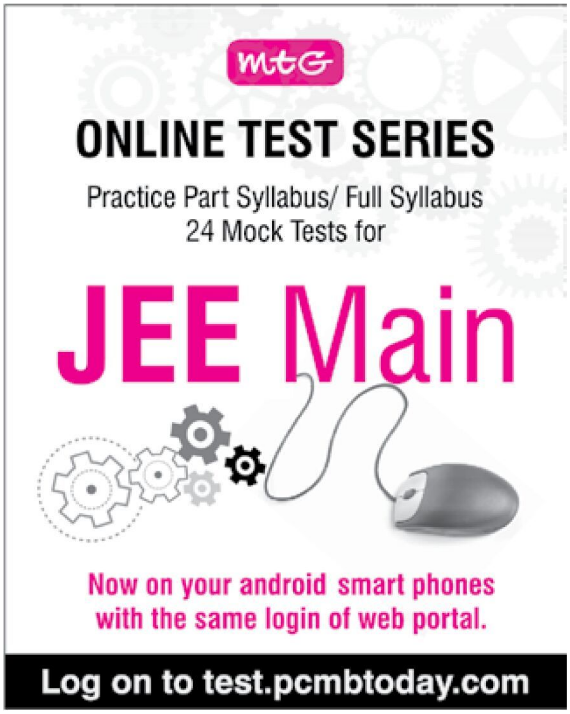
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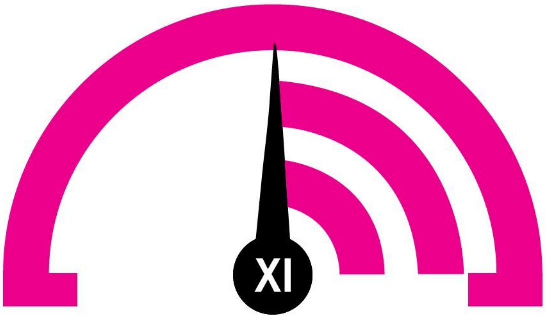


This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Straight Lines

Time Taken : 60 Min.

Only One Option Correct Type

1. The equation of a line passing through $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta + y\,\text{cosec}\,\theta = a$, is

(a) $x\cos\theta - y\sin\theta = a\cos\theta$

(b) $x\cos\theta - y\sin\theta = a\cos 2\theta$

(c) $x\cos\theta - y\sin\theta = -a\cos 2\theta$

(d) None of these

2. If the points (1, 2) and (3, 4) were to be on the same side of the line $3x - 5y + a = 0$, then

(a) $7 < a < 11$ (b) $a = 7$

(c) $a = 1$ (d) $a < 7$ or $a > 11$

3. A vertex of an equilateral triangle is (2, 3) and the opposite side is $x + y = 2$. The equations of other sides are

(a) $(2\pm\sqrt{3})x - y = 1\pm 2\sqrt{3}$

(b) $(2\pm\sqrt{3})x - y = 1\mp 2\sqrt{3}$

(c) $(1\pm 2\sqrt{3})x - y = 2\pm 2\sqrt{3}$

(d) $(2\pm\sqrt{3})x + y = 1\pm 2\sqrt{3}$

4. Find the transformed equation of the straight line $2x - 3y + 5 = 0$, when the origin is shifted to the point $(3, -1)$ after translation of axes.

(a) $2x - 3y + 14 = 0$ (b) $2x - 5y + 14 = 0$

(c) $2x - 3y + 15 = 0$ (d) $x - 4y + 14 = 0$

5. The orthocentre of the triangle formed by the points (0, 0), (4, 0) and (3, 4) is

(a) (2, 0) (b) (3/2, 2)

(c) (3/4, 3) (d) (3, 3/4)

6. The product of the perpendiculars drawn from the point (1, 2) to the pair of lines $x^2 + 4xy + y^2 = 0$ is

(a) $\frac{9}{4}$ (b) $\frac{3}{4}$

(c) $\frac{9}{16}$ (d) None of these

One or More Than One Option(s) Correct Type

7. Equations of the diagonals of a rectangle are $y + 8x - 17 = 0$ and $y - 8x + 7 = 0$. If the area of the rectangle is 8 sq. units, then the equation of the sides of the rectangle is/are

(a) $x = 1$ (b) $x + y = 1$

(c) $y = 9$ (d) $x - 2y = 3$

8. If (a, a) is not outside of lines $|x + y| = 2$, then

(a) $|a| \in [0, 1]$ (b) $|a| \in [0, 2]$

(c) $[|a|] \in \{0, 1, 2\}$ (d) $[|a|] = \{0, 1\}$

9. Which of the following line is parallel to an angle bisector of pair of lines $x^2 - y^2 - 4y - 4 = 0$?

(a) $x + 1 = 0$ (b) $y - 2 = 0$

(c) $x + y + 2 = 0$ (d) $x - 1 = 0$

EXAM ALERT 2023

Exam	Date
JEE Main Session 2	6th, 8th, 10th, 11th and 12th April
WB JEE	30th April
CUET	21st to 31st May
MHT CET PCM	9th to 13th May
BITSAT	Session - 1 : 22nd to 26th May
	Session - 2 : 18th to 22nd June
JEE Advanced	4th June

10. If the line $y = mx$ is one of the bisector of the lines $x^2 + 4xy - y^2 = 0$, then the value of m is equal to

(a) $\frac{-1+\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{2}$

(c) $\frac{-1-\sqrt{5}}{2}$ (d) $\frac{1-\sqrt{5}}{2}$

11. All points inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy

(a) $3x + 2y \geq 0$ ((b) $2x + y - 13 \geq 0$

(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$

12. If the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$, then

(a) $ax = by$ (b) $bx = ay$

(c) $x^2 - y^2 = 2(ax + by)$ (d) P can be (a, b).

13. ABC is an equilateral triangle whose centroid is origin and base BC is along the line $11x + 60y = 122$, then

(a) area of the triangle is numerically equal to the perimeter.

(b) area of triangle is numerically double the perimeter.

(c) area of triangle is numerically three times the perimeter.

(d) area of triangle is $12\sqrt{3}$ sq units.

Comprehension Type

Paragraph for Q. No. 14 to 15

A straight line L with negative slope passes through (1, 4) and meets the coordinate axes at A and B.

14. What will be the value of the sum of the intercepts made by the line L on the coordinates axes if x intercept is 3.

(a) 12 (b) 9 (c) 8 (d) 6

15. If the area of the triangle OAB is 8 units then the equation of the line L is

(a) $8x + y = 16$ (b) $16x + y = 32$

(c) $4x + y = 8$ (d) $8x + y = 8$

Matrix Match Type

16. Match the following :

	Column-I		Column-II
(P)	The coordinates of the points P and Q on the line $x + 5y = 13$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$ are	(1)	(3, 1), (–7, 11)
(Q)	The coordinates of the point on the line $x + y = 4$, which are at a unit distance from the line $4x + 3y - 10 = 0$ are	(2)	$\left(-\frac{1}{3}, \frac{11}{3}\right), \left(\frac{4}{3}, \frac{7}{3}\right)$
(R)	The coordinates of the point on the line joining $A(-2, 5)$ and $B(3, 1)$ such that $AP = PQ = QB$ are	(3)	$\left(1, \frac{12}{5}\right), \left(-3, \frac{16}{5}\right)$

	P	Q	R
(a)	1	2	3
(b)	2	3	1
(c)	3	1	2
(d)	3	2	1

Numerical Answer Type

17. If pair of lines $x^2 - 4pxy - y^2 = 0$ and $x^2 + 6qxy - y^2 = 0$ is such that each pair bisects the angle between the other pair, then pq equals ____________.

18. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ______.

19. The area enclosed by the graphs of $|x + y| = 2$ and $|x| = 1$ (in sq. units) is

20. If the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents a pair of straight lines, then the value of $221\lambda^2 + 361\lambda + 582$ is equal to

◆ ◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

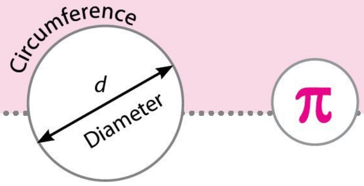
Happy
Pi DAY!

3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172 5359408128
π

March 14 marks **Pi Day**, an annual celebration of the mathematical symbol **Pi**. **Pi Day** was first organised by **Larry Shaw** in 1988. The earliest known use of the Greek letter π came in 1706. The purpose of **Pi Day** is simply to celebrate the important and mysterious number.

What is Pi?

Pi is the mathematical constant whose value is the ratio of a circle's circumference to its diameter. The first calculation of π was done by Archimedes of Syracuse.


Circumference
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Diameter
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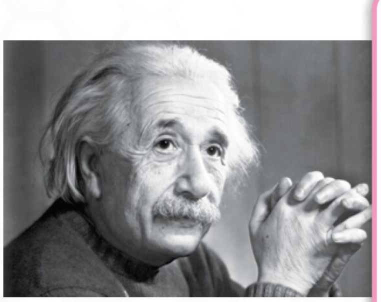


Albert Einstein's Birthday
March 14, 1879

In addition to it being **Pi Day**, it is also a significant date in the lives of two renowned scientists! On this day we commemorate the birth of Albert Einstein and the passing of Stephen Hawking. Both were aged 76 when they died. Is this Cosmic or Coincidence?

Stephen Hawking's Death Anniversary
March 14, 2018

CBSE warm-up!

Exam on 11th March 2023

CLASS-XII

Practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Practice Paper 2022-23

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- *(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
- *(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
- *(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
- *(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
- *(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
- *(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If $A=\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix, then the value of k is

(a) 4 (b) −4 (c) ±4 (d) 0

2. If $A=\begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, then (adj A)′ is equal to

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

3. $\vec{a}\cdot(\vec{b}+\vec{c})$ is equal to

(a) $\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ (b) $\vec{a}+\vec{b}\cdot\vec{c}$

(c) $\vec{a}\cdot\vec{b}+\vec{c}$ (d) $\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}\cdot\vec{c}$

4. If $y = 5\cos x - 3\sin x$, then $\dfrac{d^2y}{dx^2}$ is equal to

(a) $-y$ (b) y (c) $25y$ (d) $9y$

5. Evaluate : $\displaystyle\int_0^{\pi/2} x\cos x\,dx$

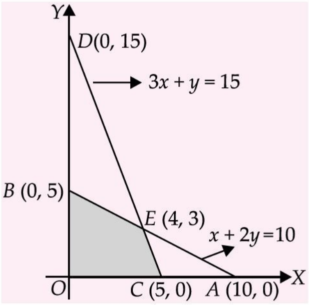
(a) $\dfrac{\pi}{6}-1$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}-1$ (d) $\dfrac{\pi}{3}-2$

6. Solve the differential equation $\dfrac{dy}{dx}=\left(\dfrac{y}{x}\right)^{1/3}$.

(a) $y^{1/3}-x^{-1/3}=c$ (b) $y^{2/3}-x^{2/3}=c$

(c) $y^{-1/3}-x^{-1/3}=c$ (d) $y^{2/3}-x^{-2/3}=c$

7. Maximise $Z = 3x + 2y$, if the shaded region is the feasible region.



(a) Max. $Z = 10$ (b) Max. $Z = 15$
(c) Max. $Z = 18$ (d) none of these

8. Find the value of $(\vec{u}\cdot\hat{i})^2+(\vec{u}\cdot\hat{j})^2+(\vec{u}\cdot\hat{k})^2$.

(a) $|\vec{u}|^2$ (b) $|\vec{u}|$ (c) 0 (d) 1

9. Evaluate : $\int |x|\,dx$

(a) $I=\dfrac{|x|}{2}+C$ (b) $I=\dfrac{x|x|}{2}+C$

(c) $I=\dfrac{x^2|x|}{2}+C$ (d) $I=\dfrac{x}{2}+C$

10. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then $\alpha =$

(a) ± 2 (b) $\pm\sqrt{5}$ (c) ± 1 (d) $\pm\sqrt{7}$

11. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).
Let $F = 4x + 6y$ be the objective function. The minimum value of F occurs at

(a) (0, 2) only (b) (3, 0) only
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only
(d) any point on the line segment joining the points (0, 2) and (3, 0)

12. If $e\begin{bmatrix} e^x & e^y \\ e^y & e^x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then the values of x and y are respectively

(a) $-1, -1$ (b) 1, 1 (c) 0, 1 (d) 1, 0

13. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

14. If the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, then find x.

(a) 1 (b) -2 (c) -1 (d) 0

15. The general solution of the differential equation $\dfrac{dy}{dx}=1-x+y-xy$ is

(a) $\log|1+y|=x-\dfrac{x^2}{2}+C$

(b) $\log|1+y|=x^2-\dfrac{x}{2}+C$

(c) $\log|1+y|=x^2-\dfrac{x}{3}+C$

(d) $\log|1+y|=\dfrac{x^3}{3}-x+C$

16. Find the order of the differential equation whose general solution is given by
$y = (A + B)\cos(x + C) + De^x$.

(a) 4 (b) 5 (c) 3 (d) 2

17. If the vector $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$, then the magnitude of $\vec{a}$ is

(a) 6 (b) 7 (c) 9 (d) 5

18. The direction cosines of the line $\dfrac{4-x}{2}=\dfrac{y}{6}=\dfrac{1-z}{3}$ are

(a) $\dfrac{2}{7},\dfrac{3}{7},\dfrac{6}{7}$ (b) $\dfrac{1}{7},\dfrac{2}{7},\dfrac{3}{7}$

(c) $\dfrac{2}{7},\dfrac{-6}{7},\dfrac{3}{7}$ (d) $\dfrac{1}{7},\dfrac{-6}{7},\dfrac{-2}{7}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A) : The domain of the function $\cos^{-1}(2x - 1)$ is [0, 1]
Reason (R) : The domain of the function $\cos^{-1} x$ is $[0, \pi]$

20. Assertion (A) : The two lines with direction cosines $\left(\dfrac{12}{13},\dfrac{-3}{13},\dfrac{-4}{13}\right),\left(\dfrac{4}{13},\dfrac{12}{13},\dfrac{3}{13}\right)$ are mutually perpendicular.
Reason (R) : The two lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are mutually perpendicular iff $l_1l_2 + m_1m_2 + n_1n_2 = 0$.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the value of $\left[\dfrac{\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)-3\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)}{\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)-3\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)}\right]$,
where $[\cdot]$ is greatest integer function.

22. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3)$.

OR

If $\vec{a}=-3\hat{i}+n\hat{j}+4\hat{k}$ and $\vec{b}=-2\hat{i}+4\hat{j}+p\hat{k}$ are collinear, then find the value of n and p.

23. Determine the value of the constant k so that the function

$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\ \dfrac{2x+1}{x-1}, & \text{if } 0\le x\le 1\end{cases}$$

is continuous at $x = 0$.

24. Find the value of x for which function

$f(x)=\sum_{k=1}^{5}(x-k)^2$ assumes minimum value.

25. Find the direction cosines of the line joining $A(0, 7, 10)$ and $B(-1, 6, 6)$.

OR

If the position vector $\vec{a}$ of a point $(12, n)$ is such that $|\vec{a}|=13$, then find the value of n.

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

27. Evaluate : $\int x^2(ax+b)^{-2}dx$

OR

Evaluate : $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx$

28. Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below:
$2x + 4y \le 8,\ 3x + y \le 6,\ x + y \le 4;\ x \ge 0,\ y \ge 0$

OR

Maximise $z = 8x + 9y$ subject to the constraints given below :
$2x + 3y \le 6,\ 3x - 2y \le 6,\ y \le 1;\ x, y \ge 0$

29. Find the solution of the differential equation $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$ satisfying $y(1) = 1$.

OR

Show that $y=cx+\frac{a}{c}$ is a solution of differential equation $y=x\frac{dy}{dx}+\frac{a}{\frac{dy}{dx}}$.

30. Evaluate : $\int\frac{dx}{\sqrt{1-2x-x^2}}$

31. Evaluate : $\int\frac{x^2+9}{x^4+81}dx$

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Using integration, find the area of the region bounded by the curves :
$y = |x + 1| + 1,\ x = -3,\ x = 3$ and $y = 0$.

33. Find the distance between the lines l_1 and l_2 given by
$l_1 : \vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$
$l_2 : \vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$.

SAMURAI SUDOKU

ANSWER - FEBRUARY 2023

8	5	9	6	3	7	2	1	4				9	6	3	5	7	1	4	8	2
2	6	1	5	4	8	9	7	3				7	5	8	2	9	4	1	3	6
7	3	4	1	2	9	5	6	8				4	1	2	3	8	6	9	5	7
6	4	8	7	9	3	1	2	5				5	8	7	9	2	3	6	1	4
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1	7	3	2	6	5	4	8	9				3	2	6	1	4	7	8	9	5
9	8	6	4	7	1	3	5	2	4	9	6	8	7	1	6	5	2	3	4	9
4	1	7	3	5	2	8	9	6	1	5	7	2	3	4	7	1	9	5	6	8
3	2	5	9	8	6	7	4	1	3	2	8	6	9	5	4	3	8	7	2	1
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						1	8	9	6	3	2	5	4	7						
						5	7	3	9	8	4	1	6	2						
8	5	2	6	3	4	9	1	7	2	6	3	4	5	8	3	1	6	2	9	7
3	9	1	7	2	8	4	6	5	8	7	1	3	2	9	4	7	8	1	5	6
7	4	6	9	1	5	2	3	8	5	4	9	7	1	6	5	2	9	3	8	4
5	3	4	2	8	6	7	9	1				2	6	7	9	8	3	5	4	1
9	6	8	5	7	1	3	4	2				9	3	4	6	5	1	8	7	2
2	1	7	4	9	3	8	5	6				1	8	5	7	4	2	6	3	9
6	8	9	3	5	7	1	2	4				8	9	3	2	6	4	7	1	5
4	7	3	1	6	2	5	8	9				5	4	2	1	3	7	9	6	8
1	2	5	8	4	9	6	7	3				6	7	1	8	9	5	4	2	3

OR

If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$ where $\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.

34. Express the following matrix as the sum of a symmetric matrix and a skew-symmetric matrix and verify your result : $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

35. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \dfrac{x-1}{x-2}$, then show that f is one-one and onto.

OR

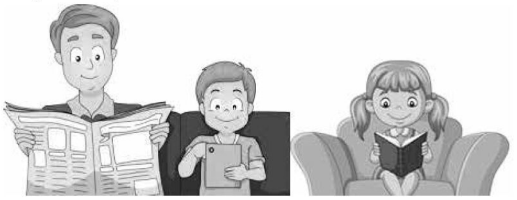
Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case-Study 1 :** Read the following passage and answer the questions given below.

On a holiday, a father gave a puzzle from a newspaper to his son Ravi and his daughter Priya. The probability of solving this specific puzzle independently by Ravi and Priya are $\frac{1}{4}$ and $\frac{1}{5}$ respectively.



(i) Find the chance that both Ravi and Priya solved the puzzle.

(ii) Find the probability that puzzle is solved by Ravi but not by Priya.

(iii) Find the probability that puzzle is solved.

OR

Find the probability that exactly one of them solved the puzzle.

37. **Case-Study 2 :** Read the following passage and answer the questions given below.

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs ₹ 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to ₹ 1200 per hour. Assume the speed of the train as v km/h.

(i) Given that the fuel cost per hour is k times the square of the speed the train generates in km/h, find the value of k.

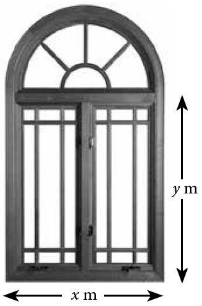
(ii) If the train has travelled a distance of 500 km, then find the function of total cost of running the train.

(iii) Determine the most economical speed to run the train.

OR

Determine the fuel cost for the train to travel 500 km at the most economical speed.

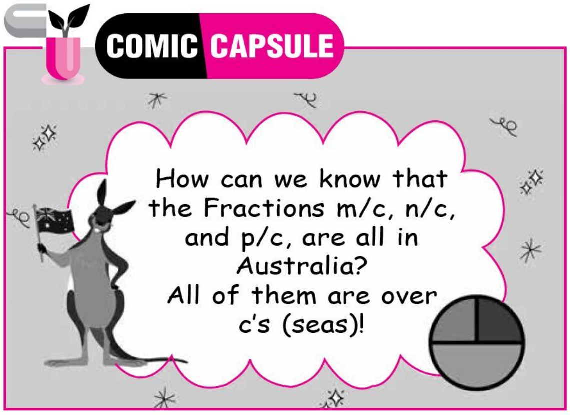
38. **Case-Study 3 :** Read the following passage and answer the questions given below.



Rohan, a student of class XII, visited his uncle's flat with his father. He observed that the window of the house is in the form of a rectangle surmounted by a semicircular opening having perimeter 10 m as shown in the figure.

(i) Find the area (A) of the window.

(ii) What is the maximum area of the window ?


COMIC CAPSULE
How can we know that the Fractions m/c, n/c, and p/c, are all in Australia?
All of them are over c's (seas)!

SOLUTIONS

1. **(c) :** $\because A$ is singular matrix.

$\therefore \quad |A| = 0$

$$\Rightarrow \begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix} = 0 \Rightarrow 2k^2 - 32 = 0 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$$

2. **(c) :** We know that, (adj A)$'$ = cofactor matrix of A

Here, cofactor matrix of $A = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix} = (\text{adj } A)'$

3. **(a) :** $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ (By distributive law)

4. **(a) :** We have, $y = 5\cos x - 3\sin x$

$$\Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$$

5. **(c) :** $\displaystyle\int_0^{\pi/2} x\cos x\, dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\sin x\, dx$

[Integrating by parts]

$$= \frac{\pi}{2} + [\cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$$

6. **(b) :** We have, $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^{1/3} \Rightarrow y^{-1/3}\, dy = x^{-1/3}\, dx$

Integrating both sides, we get

$$\frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + k \Rightarrow y^{2/3} - x^{2/3} = \frac{2}{3}k = c \text{ (say)}$$

Hence, required solution is $y^{2/3} - x^{2/3} = c$

7. **(c) :** Let us find the value of Z at corner points of the feasible region.

Clearly, $Z(O) = 0$; $Z(C) = 3 \times 5 = 15$

$Z(E) = 3 \times 4 + 2 \times 3 = 18$ and $Z(B) = 2 \times 5 = 10$

Thus, maximum value of Z is 18, at point $E(4, 3)$.

8. **(a) :** Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow (\vec{u}\cdot\hat{i})^2 = x^2$

Similarly, $(\vec{u}\cdot\hat{j})^2 = y^2$ and $(\vec{u}\cdot\hat{k})^2 = z^2$

$$\therefore \quad (\vec{u}\cdot\hat{i})^2 + (\vec{u}\cdot\hat{j})^2 + (\vec{u}\cdot\hat{k})^2 = x^2 + y^2 + z^2 = |\vec{u}|^2$$

9. **(b) :** Let $I = \int |x| \cdot 1\, dx$

$$= |x|x - \int \frac{|x|}{x} x\, dx + K = x|x| - \int |x|\, dx + K$$

$$\Rightarrow \quad I = x|x| - I + K \Rightarrow 2I = x|x| + K$$

$$\Rightarrow \quad I = \frac{x|x|}{2} + C \quad \left[\text{where } \frac{K}{2} = C\right]$$

10. **(d) :** Given, $|A^3| = 27 \Rightarrow |A|^3 = 27$

$$\Rightarrow \quad |A| = 3 \Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 3$$

$$\Rightarrow \quad \alpha^2 - 4 = 3 \Rightarrow \alpha^2 = 7 \Rightarrow \alpha = \pm\sqrt{7}$$

11. **(d) :** Construct the following table of values of objective function :

Corner Points	Value of $F = 4x + 6y$	
(0, 2)	$4 \times 0 + 6 \times 2 = 12$	$\}\leftarrow$ Minimum
(3, 0)	$4 \times 3 + 6 \times 0 = 12$	
(6, 0)	$4 \times 6 + 6 \times 0 = 24$	
(6, 8)	$4 \times 6 + 6 \times 8 = 72$	
(0, 5)	$4 \times 0 + 6 \times 5 = 30$	

Since the minimum value (F) = 12 occurs at two distinct corner points, it occurs at every point of the segment joining these two points.

12. **(a) :** $e\begin{bmatrix} e^x & e^y \\ e^y & e^x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} e^{x+1} & e^{y+1} \\ e^{y+1} & e^{x+1} \end{bmatrix} = \begin{bmatrix} e^0 & e^0 \\ e^0 & e^0 \end{bmatrix}$

$\Rightarrow \quad e^{x+1} = e^0$ and $e^{y+1} = e^0 \Rightarrow x = -1$ and $y = -1$

13. **(b) :** We have, $P(A) = 1/4$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ ($\because$ A, B are independent)

$\Rightarrow \quad 1/4 + P(B) - (1/4)\,P(B) = 2P(B) - 1/4$ (Given)

$\Rightarrow \quad P(B) = 2/5$

14. **(c) :** Given that matrix A is singular $\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0 \Rightarrow 1(-6-2) + 2(-3-x) + 3(2-2x) = 0$$

$\Rightarrow \quad -8 - 6 - 2x + 6 - 6x = 0 \Rightarrow -8x - 8 = 0 \Rightarrow x = -1$

15. **(a) :** We have $\dfrac{dy}{dx} = (1-x)(1+y) \Rightarrow \dfrac{dy}{1+y} = (1-x)dx$

$$\Rightarrow \int \frac{dy}{1+y} = \int (1-x)dx \Rightarrow \log|1+y| = x - \frac{x^2}{2} + C$$

16. **(c) :** Given $y = (A + B)\cos(x + C) + De^x$

or $\quad y = k\cos(x + C) + De^x$

Now order of a differential equation is same as the number of arbitrary unknowns present in the solution. Hence order of differential equation is 3.

17. **(b) :** Here, $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ $\therefore$ Its magnitude = $|\vec{a}|$

$$= \sqrt{3^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7.$$

18. (c) : The given equation of line is

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3} \qquad \text{...(i)}$$

$$\Rightarrow \frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3} \Rightarrow \frac{x-4}{2}=\frac{y}{-6}=\frac{z-1}{3}$$

Now, as $\sqrt{2^2+(-6)^2+3^2}=7$

$\therefore$ D.c's. of (i) are $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$.

19. (c) : We know, $0 \le \cos^{-1}(2x-1) \le \pi$

$\Rightarrow -1 \le 2x-1 \le 1 \Rightarrow 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$

$\therefore$ Domain of $\cos^{-1}(2x-1) = [0, 1]$

Also, reason is not true as domain of $\cos^{-1}x$ is $[-1, 1]$.

$\therefore$ A is true but Reason (R) is false.

20. (a) : For two lines, having direction cosines $\left(\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}\right)$ and $\left(\frac{4}{13}, \frac{12}{13}, \frac{3}{13}\right)$, we obtain

$$l_1l_2 + m_1m_2 + n_1n_2$$

$$=\frac{12}{13}\times\frac{4}{13}+\left(\frac{-3}{13}\right)\times\frac{12}{13}+\left(\frac{-4}{13}\right)\times\frac{3}{13}=0$$

Therefore, the lines are perpendicular.

So, both A and R are true and R is the correct explanation of A.

21. $$\left[\frac{\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-3\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)-3\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)}\right]=\left[\frac{\frac{\pi}{4}-3\left(\frac{\pi}{3}\right)}{\frac{\pi}{4}-3\left(\frac{\pi}{6}\right)}\right]$$

$$=\left[\frac{\frac{\pi}{4}-\pi}{\frac{\pi}{4}-\frac{\pi}{2}}\right]=3$$

$$\left[\because \text{ Principal value of } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}, \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}, \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4} \text{ and } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}\right]$$

22. Any point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}=\lambda(\text{say})$ is of the form $Q(3\lambda-2, 2\lambda-1, 2\lambda+3)$ ...(i)

Now, distance PQ, where P is $(1, 3, 3)$, is 5.

So, $(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2=5^2$

$\Rightarrow 9\lambda^2+9-18\lambda+4\lambda^2+16-16\lambda+4\lambda^2=25$

$\Rightarrow 17\lambda^2-34\lambda=0 \Rightarrow 17\lambda(\lambda-2)=0$

$\Rightarrow \lambda=0$ or $\lambda=2$

Putting values of λ in (i), the required points are $(-2, -1, 3)$ and $(4, 3, 7)$.

OR

We have, $\vec{a}$ and $\vec{b}$ are collinear.

$\therefore \vec{a}=\lambda\vec{b} \Rightarrow -3\hat{i}+n\hat{j}+4\hat{k}=\lambda(-2\hat{i}+4\hat{j}+p\hat{k})$

$\Rightarrow \lambda=\frac{3}{2}$

Also, $n=4\lambda \Rightarrow n=4\times\frac{3}{2}=6$

And, $\lambda p=4 \Rightarrow p=4\times\frac{2}{3}=\frac{8}{3}$

23. We have, L.H.L. (at $x=0$)

$$=\lim_{x\to0^-} f(x)=\lim_{x\to0}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$$

$$=\lim_{x\to0}\frac{1+kx-1+kx}{x\left(\sqrt{1+kx}+\sqrt{1-kx}\right)}$$

$$=\lim_{x\to0}\frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}}=\frac{2k}{\sqrt{1}+\sqrt{1}}=\frac{2k}{2}=k$$

R.H.L. (at $x=0$)

$$=\lim_{x\to0^+} f(x)=\lim_{x\to0}\frac{2x+1}{x-1}=-1 \text{ and } f(0)=-1$$

Since $f(x)$ is continuous at $x=0$. $\therefore k=-1$.

24. We have,

$f(x)=(x-1)^2+(x-2)^2+(x-3)^2+(x-4)^2+(x-5)^2$

$\Rightarrow f'(x)=2[x-1+x-2+x-3+x-4+x-5]$

$=2(5x-15)$

$f'(x)=0 \Rightarrow 2(5x-15)=0 \Rightarrow x=3$

Also, $f''(x)=10>0$, for all x.

$\therefore f(x)$ is minimum for $x=3$.

25. Direction ratios of AB are

$(-1-0, 6-7, 6-10)$ or $(-1, -1, -4)$

Also, $\sqrt{(-1)^2+(-1)^2+(-4)^2}=3\sqrt{2}$

$\therefore$ Direction cosines are $\left(-\frac{1}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}\right)$

or $\left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}\right)$

OR

The position vector of the point $(12, n)$ is $12\hat{i}+n\hat{j}$.

$\therefore \vec{a}=12\hat{i}+n\hat{j} \Rightarrow |\vec{a}|=\sqrt{12^2+n^2}=13$ (Given)

$\Rightarrow 12^2+n^2=169 \Rightarrow n^2=25 \Rightarrow n=\pm 5$

26. Let E : 'a total of 8' and F : 'red die resulted in a number less than 4'

i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$

i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

Hence, $E \cap F = \{(5, 3), (6, 2)\}$

$P(E) = 5/36$, $P(F) = 18/36$, $P(E \cap F) = 2/36$

$\therefore$ Required probability $= P(E|F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}$$

27. Let $I = \int \frac{x^2}{(ax+b)^2} dx$

Put $ax + b = t \Rightarrow dx = \frac{1}{a} dt$

$$\therefore \quad I = \frac{1}{a^3} \int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3} \int \left(1 + \frac{b^2}{t^2} - \frac{2b}{t}\right) dt$$

$$= \frac{1}{a^3}\left(t - \frac{b^2}{t} - 2b \log t\right) + C$$

$$= \frac{1}{a^3}\left(ax + b - \frac{b^2}{ax+b} - 2b \log|ax+b|\right) + C$$

OR

Let $I = \int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

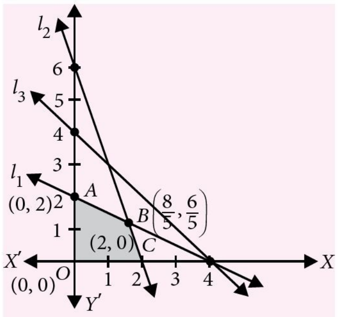
Let $a^2 \sin^2 x + b^2 \cos^2 x = t \Rightarrow (a^2 - b^2) \sin 2x \, dx = dt$

$$\therefore \quad I = \frac{1}{(a^2 - b^2)} \int \frac{1}{t} dt = \frac{1}{(a^2 - b^2)} \log|t| + C$$

$$\Rightarrow I = \frac{1}{(a^2 - b^2)} \log\left|a^2 \sin^2 x + b^2 \cos^2 x\right| + C$$

28. Let $l_1 : 2x + 4y = 8, l_2 : 3x + y = 6, l_3 : x + y = 4; x = 0, y = 0$

Solving l_1 and l_2 we get $B\left(\frac{8}{5}, \frac{6}{5}\right)$



Shaded portion $OABC$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 2)$, $B\left(\frac{8}{5}, \frac{6}{5}\right)$, $C(2, 0)$

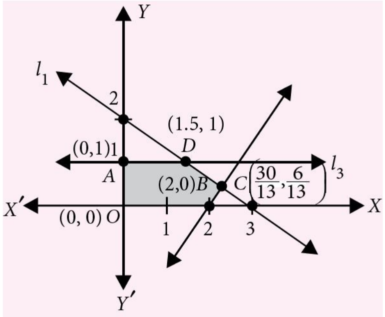
The value of objective function at these points are :

Corner Points	Value of the objective function $z = 2x + 5y$
$O(0, 0)$	$2 \times 0 + 5 \times 0 = 0$
$A(0, 2)$	$2 \times 0 + 5 \times 2 = 10$ (Maximum)
$B\left(\frac{8}{5}, \frac{6}{5}\right)$	$2 \times \frac{8}{5} + 5 \times \frac{6}{5} = 9.2$
$C(2, 0)$	$2 \times 2 + 5 \times 0 = 4$

$\therefore$ The maximum value of z is 10, which is at $A(0, 2)$.

OR

Let $l_1 : 2x + 3y = 6$, $l_2 : 3x - 2y = 6$, $l_3 : y = 1$; $x = 0$, $y = 0$



Solving l_1 and l_3, we get D (1.5, 1)

Solving l_1 and l_2, we get $C\left(\frac{30}{13}, \frac{6}{13}\right)$

Shaded portion $OADCBO$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 1)$, $D(1.5, 1)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, $B(2, 0)$.

The value of the objective function at these points are :

Corner Points	Value of the objective function $z = 8x + 9y$
O (0, 0)	$8 \times 0 + 9 \times 0 = 0$
A (0, 1)	$8 \times 0 + 9 \times 1 = 9$
D (1.5, 1)	$8 \times 1.5 + 9 \times 1 = 21$
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	$8 \times \frac{30}{13} + 9 \times \frac{6}{13} = 22.6$ (Maximum)
B (2, 0)	$8 \times 2 + 9 \times 0 = 16$

The maximum value of z is 22.6, which is at $C\left(\frac{30}{13}, \frac{6}{13}\right)$

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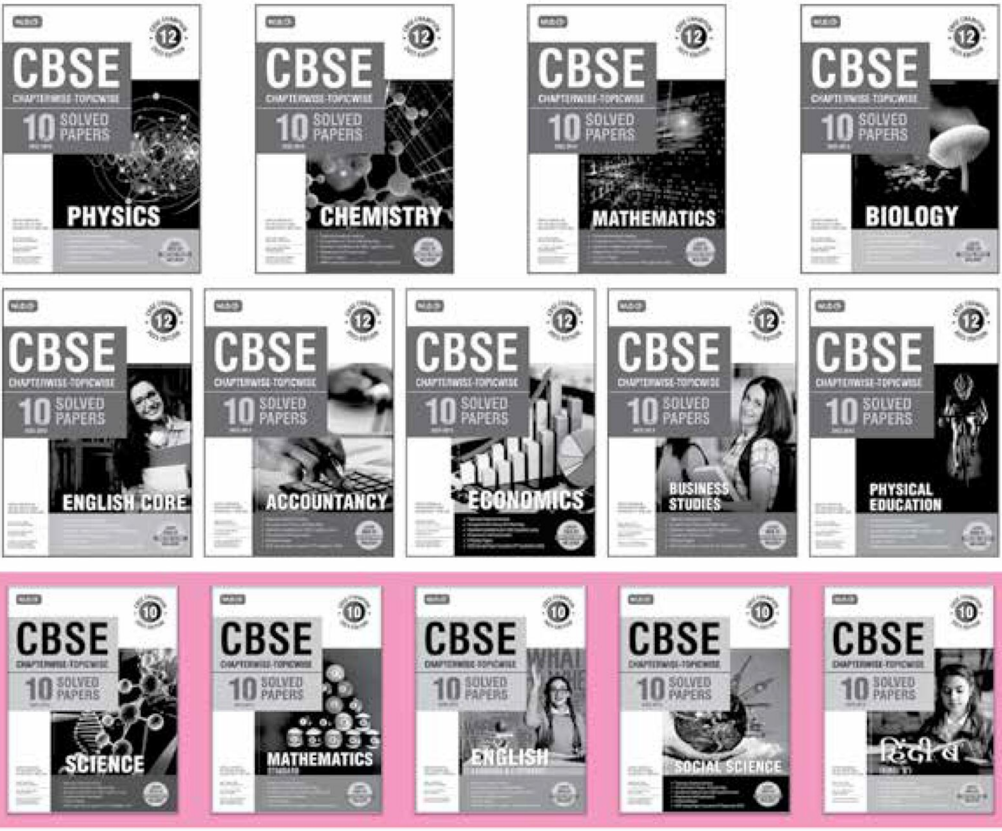
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29. Given, $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$

$\Rightarrow\ 2xydy = (x^2 + y^2 + 1)dx$

$\Rightarrow\ 2xydy - y^2dx = (x^2 + 1)dx$

$\Rightarrow\ xd(y^2) - y^2dx = (x^2 + 1)dx$

$$\Rightarrow\ \frac{xd(y^2)-y^2dx}{x^2}=\left(1+\frac{1}{x^2}\right)dx \Rightarrow d\left(\frac{y^2}{x}\right)=d\left(x-\frac{1}{x}\right)$$

Integrating both sides, we get

$$\frac{y^2}{x}=x-\frac{1}{x}+C \Rightarrow y^2 = x^2 - 1 + Cx$$

Now, given that $y(1) = 1$

$\therefore\ 1 = 1 - 1 + C\ \Rightarrow\ C = 1$

Thus, curve becomes $y^2 = x^2 - 1 + x$

OR

We have, $y=cx+\frac{a}{c}$ ...(i)

Differentiating with respect to x, we get $\frac{dy}{dx}=c$

$$\therefore\ x\frac{dy}{dx}+\frac{a}{\frac{dy}{dx}}=xc+\frac{a}{c} \qquad \left[\text{Putting } \frac{dy}{dx}=c\right]$$

$$\Rightarrow\ x\frac{dy}{dx}+\frac{a}{\frac{dy}{dx}}=y \qquad [\text{Using (i)}]$$

Hence, $y=cx+\frac{a}{c}$ is a solution of the differential equation $x\frac{dy}{dx}+\frac{a}{dy/dx}=y$.

30. Let $I=\int\frac{dx}{\sqrt{1-(x^2+2x)}}=\int\frac{dx}{\sqrt{2-(x^2+2x+1)}}$

$$=\int\frac{dx}{\sqrt{2-(1+x)^2}}=\int\frac{dx}{\sqrt{(\sqrt{2})^2-(1+x)^2}}$$

Let $1 + x = z\ \Rightarrow\ dx = dz$

$$\therefore\ I=\int\frac{dz}{\sqrt{(\sqrt{2})^2-z^2}}=\sin^{-1}\frac{z}{\sqrt{2}}+c=\sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)+c$$

31. Let $\text{I}=\int\frac{x^2+9}{x^4+81}dx\ \Rightarrow\ \text{I}=\int\frac{1+(9/x^2)}{x^2+\frac{81}{x^2}}dx$

$$\Rightarrow\ I=\int\frac{1+(9/x^2)}{x^2+\left(\frac{9}{x}\right)^2-18+18}dx=\int\frac{1+(9/x^2)}{\left(x-\frac{9}{x}\right)^2+18}dx$$

Let $x-\frac{9}{x}=t\Rightarrow\left(1+\frac{9}{x^2}\right)dx=dt$

$$\therefore\ \text{I}=\int\frac{dt}{t^2+18}\ \Rightarrow\ \text{I}=\int\frac{dt}{t^2+\left(3\sqrt{2}\right)^2}$$

$$\Rightarrow\ \text{I}=\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{t}{3\sqrt{2}}\right)+c$$

$$\Rightarrow\ \text{I}=\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right)+c$$

32. Given, $y = |x + 1| + 1$

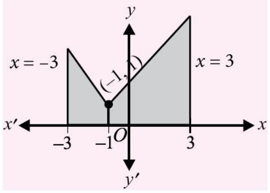
$$\therefore\ y=\begin{cases}x+2\ , & \text{if}\quad x\ge -1\\ -x\ , & \text{if}\quad x<-1\end{cases}$$

We now draw the lines : $y = 0$, $x = 3$, $x = -3$ and

$y = x + 2$ if $x \ge -1$ ...(i)

$y = -x$ if $x < -1$ ...(ii)

Lines (i) and (ii) intersect each other at $(-1, 1)$



$\therefore$ Required area

$$=\int_{-3}^{-1}(-x)dx+\int_{-1}^{3}(x+2)dx$$

$$=-\left[\frac{x^2}{2}\right]_{-3}^{-1}+\left[\frac{x^2}{2}+2x\right]_{-1}^{3}$$

$$=-\frac{1}{2}(1-9)+\frac{1}{2}(9-1)+2(3+1)=4+4+8=16 \text{ sq. units.}$$

33. Given lines are

$l_1:\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k});$

$l_2:\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

$\therefore$ We have $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k},\vec{b}_1=2\hat{i}+3\hat{j}+6\hat{k}$

and $\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k},\vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}$

So, $\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}=2\vec{b}_1\ \Rightarrow\ \vec{b}_1\,||\,\vec{b}_2$

Hence l_1 and l_2 are parallel lines.

Shortest distance between two parallel lines is,

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|\ \Rightarrow\ d=\left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\Rightarrow\ d=\left|\frac{-9\hat{i}+14\hat{j}-4\hat{k}}{7}\right|$$

$$\Rightarrow\ d=\frac{\sqrt{(-9)^2+14^2+(-4)^2}}{7}=\frac{\sqrt{293}}{7}\text{ units.}$$

OR

Here, $\vec{a}=3\hat{i}-\hat{j},\ \vec{b}=2\hat{i}+\hat{j}-3\hat{k}$

We have to express : $\vec{b}=\vec{b}_1+\vec{b}_2$, where

$\vec{b}_1\,||\,\vec{a}$ and $\vec{b}_2\perp\vec{a}$

Let $\vec{b}_1 = \lambda\vec{a} = \lambda\left(3\hat{i} - \hat{j}\right)$ and $\vec{b}_2 = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\vec{b}_2 \perp \vec{a} \Rightarrow \vec{b}_2 \cdot \vec{a} = 0$

$\Rightarrow \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\left(3\hat{i} - \hat{j}\right) = 0 \;\Rightarrow\; 3x - y = 0$ ...(i)

Now, $\vec{b} = \vec{b}_1 + \vec{b}_2$

$\Rightarrow 2\hat{i} + \hat{j} - 3\hat{k} = \lambda\left(3\hat{i} - \hat{j}\right) + \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

On comparing, we get

$$\left.\begin{array}{l} 2 = 3\lambda + x \\ 1 = -\lambda + y \end{array}\right] \Rightarrow x + 3y = 5 \qquad \text{...(ii)}$$

and $-3 = z \Rightarrow z = -3$

Solving (i) and (ii), we get $x = \frac{1}{2}, y = \frac{3}{2}$

$\therefore 1 = -\lambda + y = -\lambda + \frac{3}{2} \Rightarrow \lambda = \frac{1}{2}$

Hence, $\vec{b}_1 = \lambda\left(3\hat{i} - \hat{j}\right) = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$ and $\vec{b}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

34. Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$\therefore\; A + A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\Rightarrow\; \frac{1}{2}(A + A') = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

which is clearly a symmetric matrix and

$$A - A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\Rightarrow\; \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

which is clearly a skew symmetric matrix.

Since $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$$\therefore\; A = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

Thus A has been expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

35. Here, $f : A \to B$ is given by $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2, x_2 \neq 2$)

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$

$\Rightarrow -2x_1 - x_2 = -x_1 - 2x_2 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$

such that $f(x) = y$

$\Leftrightarrow \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x - 1$

$\Leftrightarrow xy - 2y = x - 1 \Leftrightarrow x(y - 1) = 2y - 1 \Leftrightarrow x = \frac{2y-1}{y-1}$

$\therefore f(x) = y$ when $x = \frac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto. Thus, f is one–one and onto.

OR

Given relation is $R = \{(a, b) : a \leq b^2\}$

Reflexivity: Let $a \in$ real numbers.

$aRa \Rightarrow a \leq a^2$

but if $a < 1$, then $a \nleq a^2$

For example, $a = \frac{1}{2} \Rightarrow a^2 = \frac{1}{4}$ so, $\frac{1}{2} \nleq \frac{1}{4}$

UNSCRAMBLED WORDS

FEBRUARY 2023

1-i- EXPANSION
2-h- DEVIATION
3-g- FRACTION
4-f- PARALLELEPIPED
5-a- SLOPE
6-j- PREDECESSOR
7-e- TRAPEZIUM
8-d- ALGEBRA
9-c- STATISTICS
10-b-SPHERE

Hence, R is not reflexive.
Symmetricity : $aRb \Rightarrow a \le b^2$
But then $b \le a^2$ is not true $\therefore\ aRb \not\Rightarrow bRa$

For example, $a = 2$, $b = 5$ then $2 \le 5^2$ but $5 \le 2^2$ is not true.
Hence, R is not symmetric.
Transitivity : Let $a, b, c \in$ real numbers
Considering aRb and bRc
$aRb \Rightarrow\ a \le b^2$ and $bRc \Rightarrow b \le c^2\ \Rightarrow\ a \le c^4 \not\Rightarrow aRc$
For example, if $a = 2$, $b = -3$, $c = 1$
$aRb \Rightarrow 2 \le 9$; $bRc \Rightarrow -3 \le 1$; $aRc \Rightarrow 2 \le 1$ is not true.
Hence, R is not transitive

36. Let E_1 be the event that Ravi solved the puzzle and E_2 be the event that Priya solved the puzzle.
Then, $P(E_1) = 1/4$ and $P(E_2) = 1/5$

(i) Since, E_1 and E_2 are independent events.
$\therefore$ P(both solved the puzzle) $= P(E_1 \cap E_2)$

$$= P(E_1)\cdot P(E_2) = \frac{1}{4}\times\frac{1}{5} = \frac{1}{20} = \frac{1}{20}\times 100\% = 5\%$$

(ii) P(puzzle is solved by Ravi but not by Priya)

$$= P(\bar{E}_2)P(E_1) = \left(1-\frac{1}{5}\right)\cdot\frac{1}{4} = \frac{4}{5}\cdot\frac{1}{4} = \frac{1}{5}.$$

(iii) P(puzzle is solved) $= P(E_1$ or $E_2)$
$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{1}{4}+\frac{1}{5}-\frac{1}{20} = \frac{8}{20} = \frac{2}{5}.$$

OR

P(Exactly one of them solved the puzzle)

$= P[(E_1$ and $\bar{E}_2)$ or $(E_2$ and $\bar{E}_1)]$

$= P(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)$

$= P(E_1) \times P(\bar{E}_2) + P(E_2) \times P(\bar{E}_1)$

$$= \frac{1}{4}\times\frac{4}{5}+\frac{1}{5}\times\frac{3}{4} = \frac{4}{20}+\frac{3}{20} = \frac{7}{20}. \quad [\because P(\bar{E}_1) = 1 - P(E_1)]$$

37. (i) Let F be the fuel cost per hour and v be the speed of train in km/hr.
According to question, we have,
$F \propto v^2 \Rightarrow F = kv^2$, where k is proportionality constant

$$\Rightarrow\ 48 = k(16)^2\ \Rightarrow k = \frac{3}{16}$$

(ii) Let total cost of running the train be C.

Then, $C = \frac{3}{16}v^2 t + 1200t$

Now, distance covered = 500 km $\Rightarrow$ Time $= \frac{500}{v}$ hrs

$\therefore$ Total cost of running the train for 500 km

$$= \frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)\ \Rightarrow\ C = \frac{375}{4}v + \frac{600000}{v}$$

(iii) We have, $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$

Put $\frac{dC}{dv} = 0\ \Rightarrow\ v^2 = \frac{600000\times 4}{375} = 6400 \Rightarrow v = 80$ km/h

$\frac{d^2C}{dv^2} = \frac{2\times 600000}{v^3} > 0$, for $v = 80$

$\therefore$ Most economical speed is 80 km/h.

OR

Fuel cost for running the train for 500 km

$$= \frac{3}{16}v^2\left(\frac{500}{v}\right) = \frac{375}{4}v = \frac{375}{4}\times 80 = ₹\,7500$$

38. (i) Given, perimeter of window = 10 m
$\therefore$ $x + y + y$ + perimeter of semicircle = 10

$$\Rightarrow\ x + 2y + \pi\frac{x}{2} = 10\ \Rightarrow\ y = 5 - \frac{x}{2} - \frac{\pi x}{4} \qquad \text{...(i)}$$

$$A = x\cdot y + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = x\left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\frac{\pi x^2}{4}$$

$[\because$ From (i)$]$

$$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

(ii) We have, $A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}\ \Rightarrow\ \frac{dA}{dx} = 5 - x - \frac{\pi x}{4}$

Now, $\frac{dA}{dx} = 0\ \Rightarrow\ 5 = x + \frac{\pi x}{4}$

$$\Rightarrow\ x(4+\pi) = 20 \Rightarrow\ x = \frac{20}{4+\pi}$$

$\left[\text{Clearly, } \frac{d^2A}{dx^2} < 0 \text{ at } x = \frac{20}{4+\pi}\right]$

At $x = \frac{20}{4+\pi}$

$$A = 5\left(\frac{20}{4+\pi}\right) - \left(\frac{20}{4+\pi}\right)^2\frac{1}{2} - \frac{\pi}{8}\left(\frac{20}{4+\pi}\right)^2$$

$$= \frac{100}{4+\pi} - \frac{200}{(4+\pi)^2} - \frac{50\pi}{(4+\pi)^2}$$

$$= \frac{(4+\pi)(100) - 200 - 50\pi}{(4+\pi)^2} = \frac{400 + 100\pi - 200 - 50\pi}{(4+\pi)^2}$$

$$= \frac{200 + 50\pi}{(4+\pi)^2} = \frac{50(4+\pi)}{(4+\pi)^2} = \frac{50}{4+\pi}$$

◆◆

Quiz CLUB

Unlock Your Knowledge!

1. Find the projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}\cdot\vec{b}=8$ and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$.
2. Write the number of all the one-one functions from $A = \{1, 2\ 3\}$ to itself.
3. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$, , then find the value of $a + b$.
4. If A and B are disjoint sets and A' is the complement of A, then find is the value of $B \cap A'$.
5. Find the value of x if $\cos 2x = \cos x$ for $0 \le x \le \pi$.
6. Find the value of $7! - 5!$.
7. The arithmetic mean of two numbers a and b is 3 and geometric mean is 2. Find the value of $a^2 + b^2$.
8. Find the last digit of $(1! + 2! + 3! + ... + 2005!)^{502}$.
9. Write the derivative of $f(x) = \cos x$ at $x = 0$.
10. If A and B are square matrices of the same order and if $A = A^T$, $B = B^T$, then find the value of the transpose of (ABA).
11. What is the equation of tangent to $y = -2x^2 + 3$ at $x = 1$?
12. Find the value of $\int_{e}^{e^2} \frac{1}{x}\, d.$.
13. If $0 \le A,\ B \le \frac{\pi}{2}$ and $\sin A + \cos B = 2$, then find the value of $\tan\left(\frac{A+B}{2}\right)$.
14. If $\tan(x + y) = 33$ and $x = \tan^{-1}3$, then what is the value of y?
15. Find the value of $f[f(x)]$ if $f(x) = \frac{x+2}{3x-1}$.
16. Find the value of $2b + c$, for the function $f(x) = 2x^3 + bx^2 + cx$, which holds Rolle's theorem for $x \in [-1, 1]$ at $x = 1/2$.
17. Find the probability of showing at least one head if a pair of coin is tossed once.
18. What will be value of $2y(2) - y(1)$ for $\frac{dy}{dx} + \frac{y}{x} = x^2$?
19. Find standard deviation of four observations if their mean is 3 and sum of the squares of these observations is 48.
20. If 3 out of 6 vertices of a regular hexagon are chosen at a time at random, then what will be the probability that the triangle with three vertices is an equilateral triangle?

◆◆

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

SOLUTIONS TO FEBRUARY 2023 QUIZ CLUB

1. $10C^3$
2. $6 + 11i$
3. 7 units
4. $2\cos a$
5. 671
6. $2 + \sqrt{3}$
7. 0.6
8. $\frac{4}{\sqrt{5}-1}$
9. A
10. $x = y$
11. $3f(x)$
12. $2\left(x^2y^2 + y^2z^2 + z^2x^2\right)$
13. $\frac{1}{2}$
14. A
15. $\frac{-4x}{1-x^4}$
16. $\tan^{-1}\sqrt{\frac{p}{q}}$
17. $\frac{7}{3}$
18. $\tan(y/x) = cx$
19. $\frac{5\sqrt{3}}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$
20. 90°

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Differential Equations

Time Taken : 60 Min.

Only One Option Correct Type

1. If p and q are order and degree of differential equation
$$y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{1/3} + x^2y^2 = \sin x, \text{ then}$$
(a) $p > q$ (b) $\frac{p}{q} = \frac{1}{2}$
(c) $p = q$ (d) $p < q$

2. Solve : $\left(x\sin\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - x\right)dx.$
(a) $\cos\frac{y}{x} = \log x + c$ (b) $\sin\frac{y}{x} = \log x + c$
(c) $\cot x = \log y + c$ (d) None of these

3. The differential equation of all non-vertical lines in a plane, is
(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2x}{dy^2} = 0$
(c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

4. The solution of the differential equation
$e^{-x}(y + 1)dy + (\cos^2 x - \sin 2x)y\,dx = 0$
subjected to the condition that $y = 1$ when $x = 0$ is
(a) $(y + 1) + e^x\cos^2 x = 2$
(b) $y + \log y = e^x\cos^2 x$
(c) $\log(y + 1) + e^x\cos^2 x = 1$
(d) $y + \log y + e^x\cos^2 x = 2$

5. The equation of a curve passing through (0, 1) and having gradient $\frac{-(y + y^3)}{1 + x + xy^2}$ at (x, y) is
(a) $xy + \tan^{-1} y = \frac{\pi}{2}$ (b) $xy + \tan^{-1} y = \frac{\pi}{4}$
(c) $xy - \tan^{-1} y = \frac{\pi}{2}$ (d) $xy - \tan^{-1} y = \frac{\pi}{4}$

6. If $ye^y dx = (y^3 + 2xe^y)\,dy$, $y(0) = 1$, then the value of x when $y = 0$ is
(a) -1 (b) 0 (c) 1 (d) 2

One or More Than One Option(s) Correct Type

7. If the differential equation of the family of all circles whose centre lies on the straight line $y = x$ is represented by the $P\frac{d^2y}{dx^2} + Q\frac{dy}{dx} + 1 = 0$, where P & Q are functions of x & y, then which of the following is/are true?
(a) $P = y + x$ (b) $P = y - x$
(c) $P + Q = 1 - x + y + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$
(d) $P - Q = x + y - \frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2$

8. The differential equation whose solution represents the family $y = Ae^{3x} + Be^{-7x}$ is $y_2 + \lambda y_1 + my = 0$, then
(a) $\lambda + |m| = 25$
(b) Number of prime factors of $|m|$ is 2
(c) $\lambda = 16$
(d) If λ is represented as exponent of a prime number then base is smallest prime.

9. If the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$, given that for $t = 0, x = 1$ and $\frac{dx}{dt} = 10$ is in the form $x = e^{-2t}(A\cos 3t + B\sin 3t)$ then
(a) $A + B = 5$ (b) $AB = 4$
(c) $AB = 5$ (d) None of these

10. Which of the following statement(s) is/are true?

(a) $f(x, y) = e^{y/x} + \tan\frac{y}{x}$ is a homogeneous equation.

(b) $x \ln\frac{y}{x}dx + \frac{y^2}{x}\sin^{-1}\frac{y}{x}dy = 0$ is a homogeneous differential equation.

(c) $f(x, y) = x^2 + \sin x \cos y$ is a non-homogeneous equation.

(d) $(x^2 + y^2)dx - (xy^2 - y^3)dy = 0$ is a homogeneous differential equation.

11. The equation of a curve passing through (2, 7/2) and having gradient $1 - 1/x^2$ at (x, y) is $xy = ax^2 + bx + c$, then

(a) $a = b = c$ (b) $abc = 1$

(c) $a + b + c = 3$ (d) $2a + b = 4$

12. Which of the following equation(s) is/are linear?

(a) $\frac{dy}{dx} + \frac{y}{x} = \log x$ (b) $y\left(\frac{dy}{dx}\right) + 4x = 0$

(c) $(2x + y^3)\left(\frac{dy}{dx}\right) = 3y$ (d) None of these

13. If $y = a \sin(bx + c)$ be the solution of a differential equation, then

(a) $y\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$

(b) order of differential equation is 3.

(c) degree of differential equation is 1.

(d) $y^2\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^3$

Comprehension Type

Paragraph for Q. No. 14 and 15

Differential equation $\frac{dy}{dx} = f(x)g(y)$ can be solved by separating the variables *i.e.*, $\frac{dy}{g(y)} = f(x)dx$.

14. Solution of the differential equation

$\frac{dy}{dx} + \frac{1 + y^2}{\sqrt{1 - x^2}} = 0$ is

(a) $\tan^{-1}y + \sin^{-1}x = C$ (b) $\tan^{-1}x + \sin^{-1}y = C$

(c) $\tan^{-1}y \sin^{-1}x = C$ (d) $\tan^{-1}y - \sin^{-1}x = C$

15. If $dy/dx = 1 + x + y + xy$ and $y(-1) = 0$, then $y =$

(a) $e^{\frac{(1-x)^2}{2}}$ (b) $e^{\frac{(1+x)^2}{2}} - 1$

(c) $\log(1 + x) - 1$ (d) $1 + x$

Matrix Match Type

16. Match the following :

Column - I		Column - II	
(P)	The solution of $\frac{dy}{dx} = 2^{y-x}$ is	(1)	$x^2(y + 3)^3 = e^{y+2}$
(Q)	The solution of $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0, x = 0$ is	(2)	$2^{-x} - 2^{-y} = k$
(R)	Solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is	(3)	$y = \tan\left(x + \frac{x^2}{2}\right)$
(S)	The solution of $2(y + 3) - xy\frac{dy}{dx} = 0$, given that $y(1) = -2$ is	(4)	$\tan^{-1}\left(\frac{y}{x}\right) = \log\lvert x\rvert + C$

	P	Q	R	S
(a)	2	3	1	4
(b)	3	2	4	1
(c)	3	2	1	4
(d)	2	3	4	1

Numerical Answer Type

17. If $x\frac{dy}{dx} = x^2 + y - 2$, $y(1) = 1$, then $y(-1)$ equals ___.

18. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$, when $x = e$, then value of x, when $y = \frac{1}{2}$ is $\frac{3e - 1}{2k}$. Find the value of k.

19. Order of the differential equation whose solution is $xy = ae^x + be^{-x} + c$ is____.

20. Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is ___.

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

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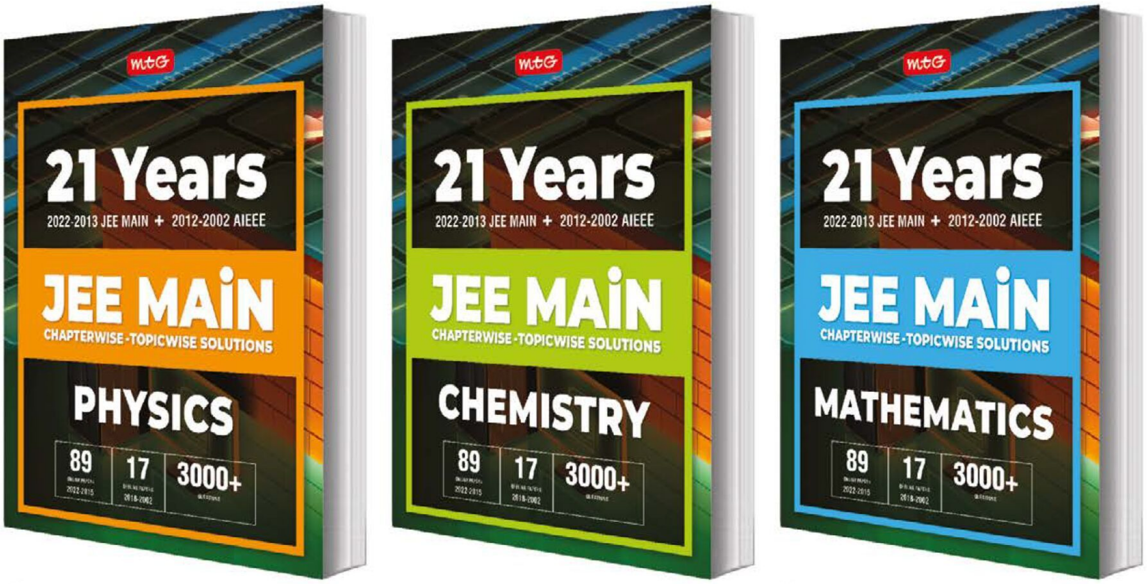
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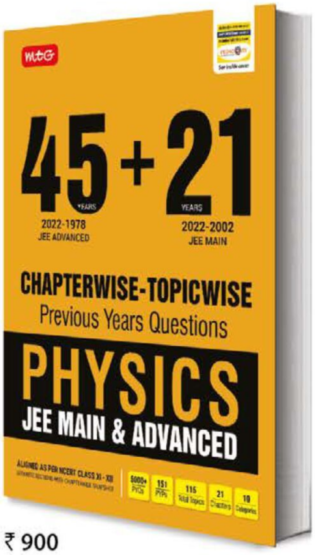
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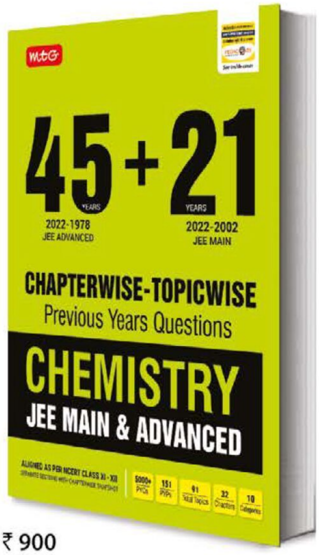
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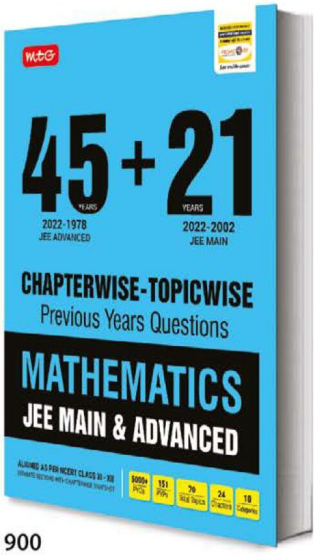
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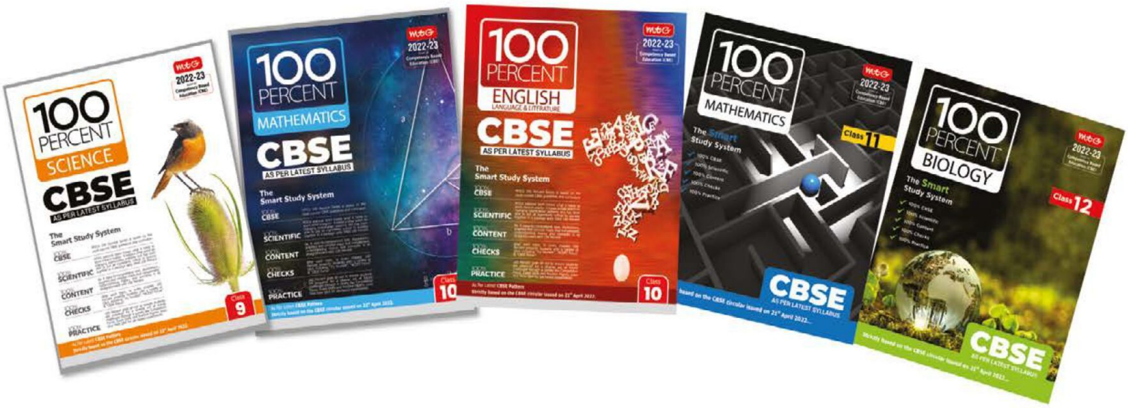
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